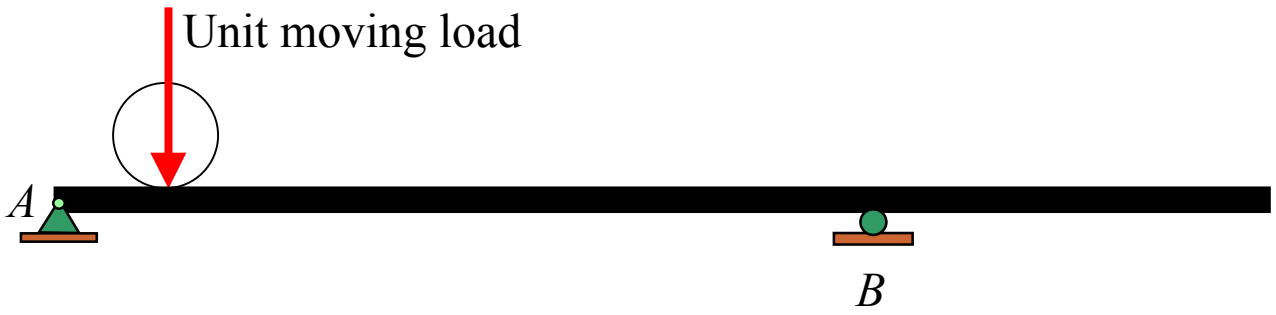
INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES

- Influence Lines for Beams
- Influence Lines for Floor Girders
- Influence Lines for Trusses
- Maximum Influence at a Point Due to a Series of Concentrated Loads
- Absolute Maximum Shear and Moment

Influence Line

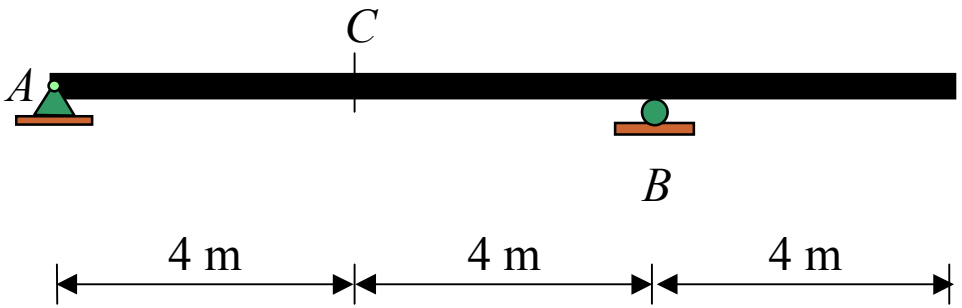


Example 6-1

Construct the influence line for

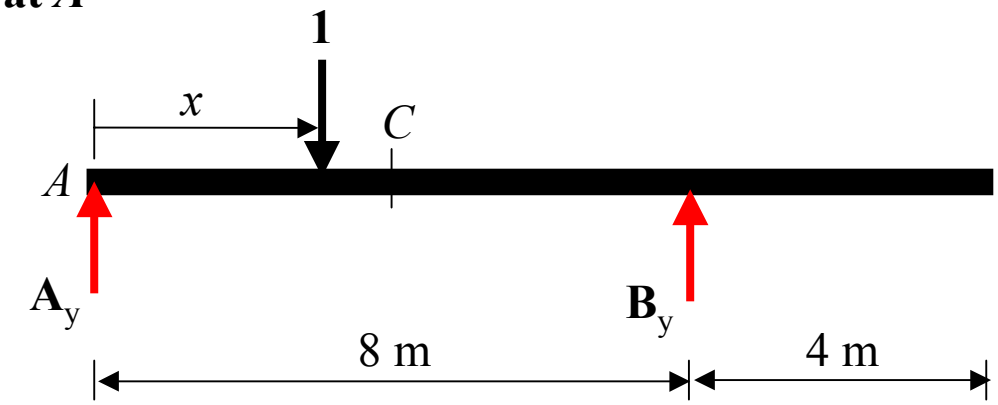
a) reaction at *A* and *B*
b) shear at point C
c) bending moment at point C
d) shear before and after support *B*
e) moment at point *B*

of the beam in the figure below.



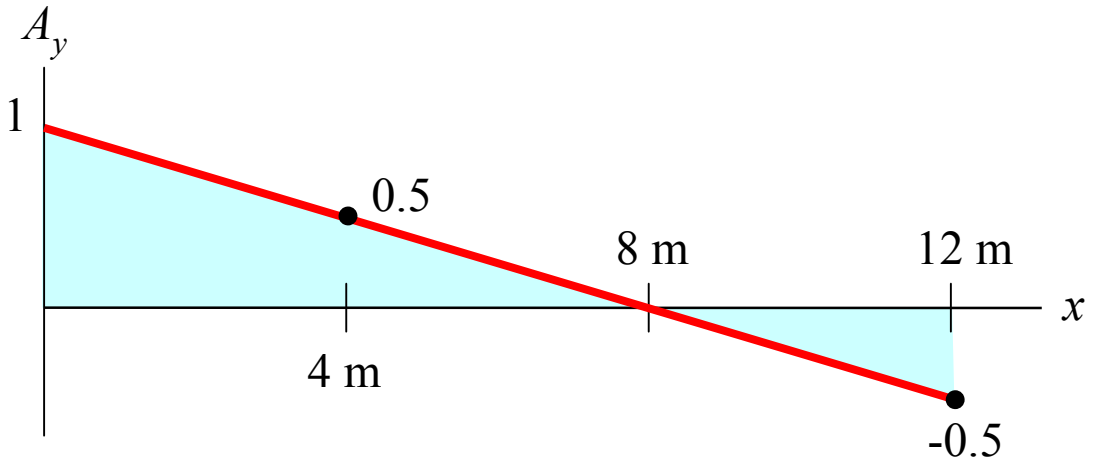
SOLUTION

- **Reaction at *A***

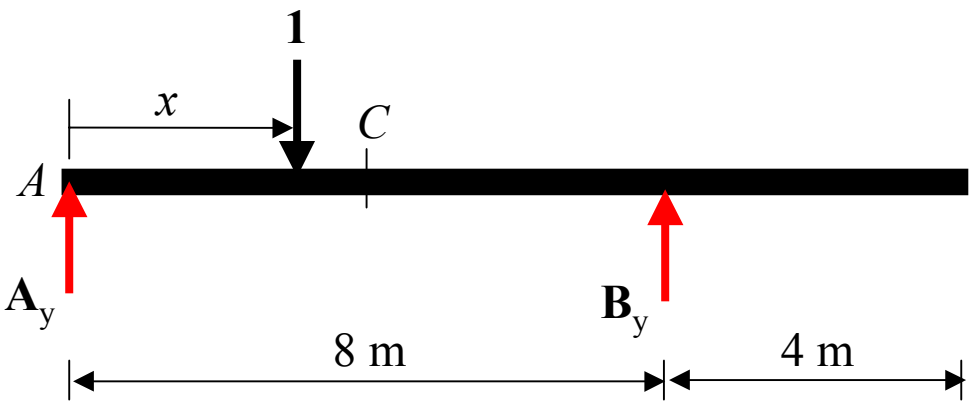


x	A_y
0	1
4	0.5
8	0
12	-0.5

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(8) + 1(8 - x) = 0, \qquad A_y = 1 - \frac{1}{8}x$$

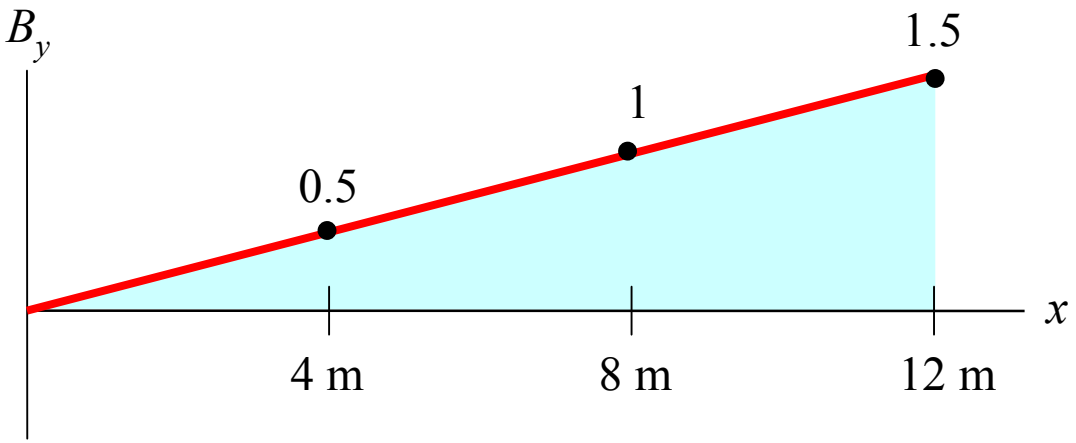


• **Reaction at *B***

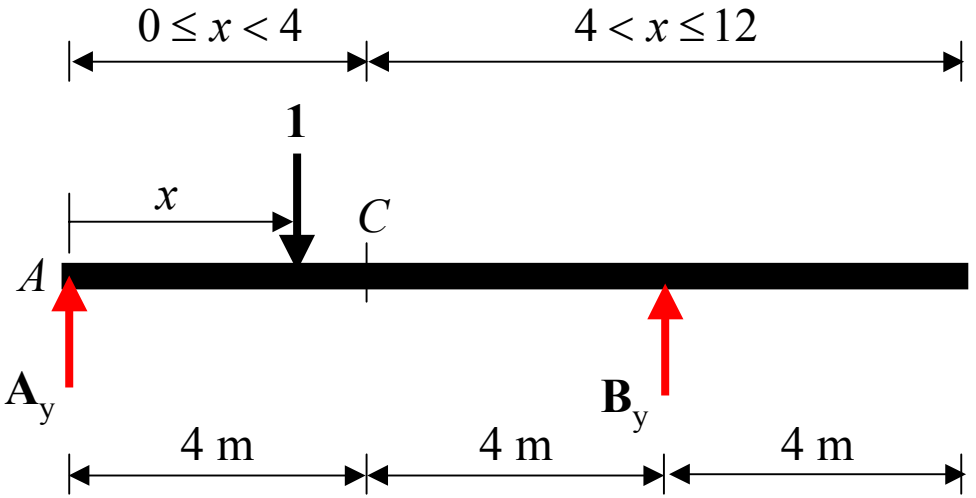


x	B_y
0	0
4	0.5
8	1
12	1.5

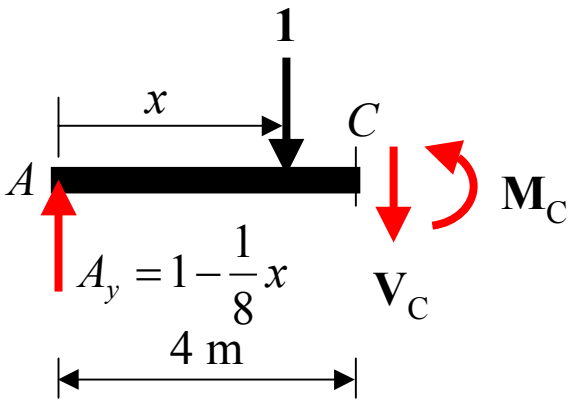
$$+\circlearrowleft \Sigma M_A = 0: \qquad B_y(8) - 1x = 0, \qquad B_y = \frac{1}{8}x$$



• **Shear at *C***



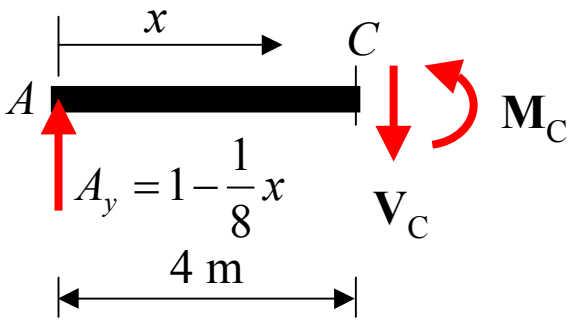
$0 \le x \le 4$



$+\uparrow \Sigma F_y = 0: \qquad 1 - \frac{1}{8}x - 1 - V_C = 0$

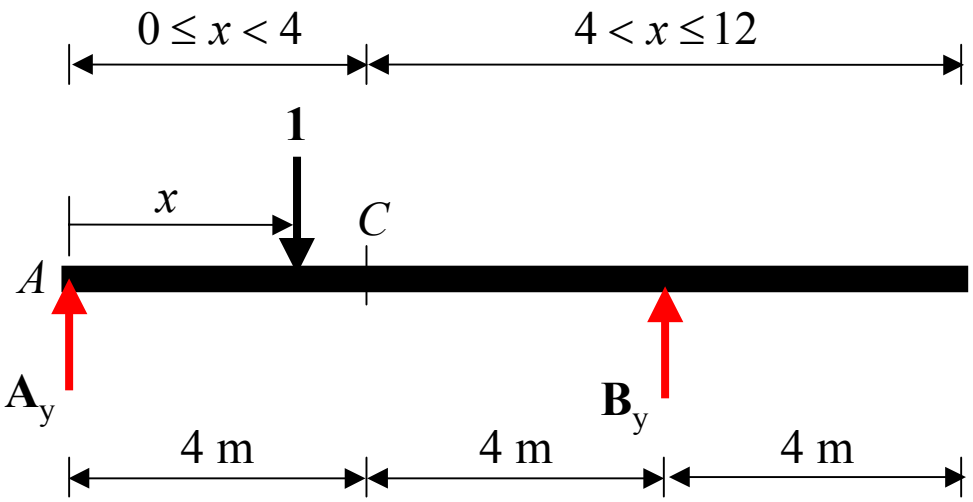
$$V_C = -\frac{1}{8}x$$

$4 < x \le 12$

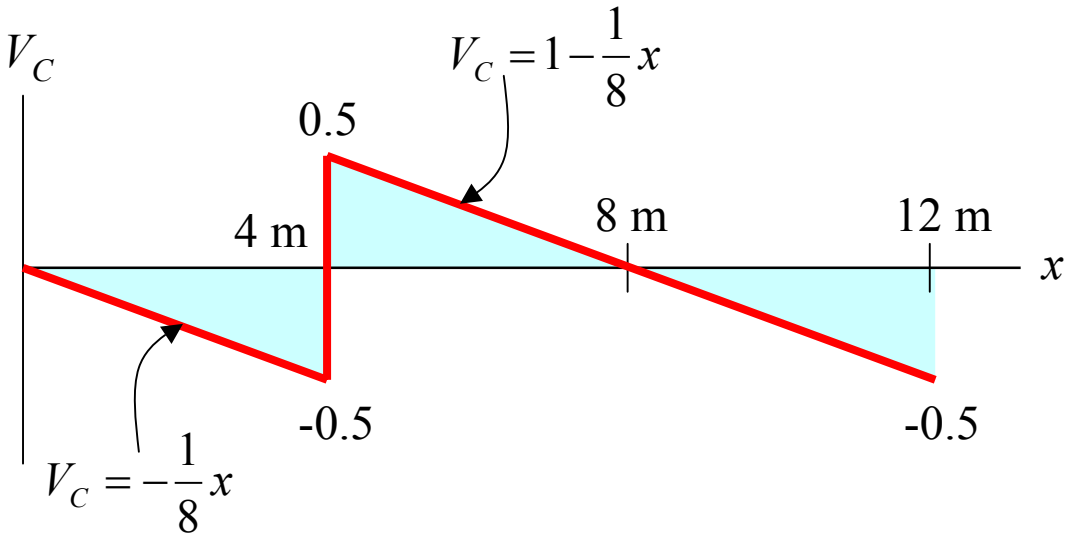


$+\uparrow \Sigma F_y = 0: \qquad 1 - \frac{1}{8}x - V_C = 0$

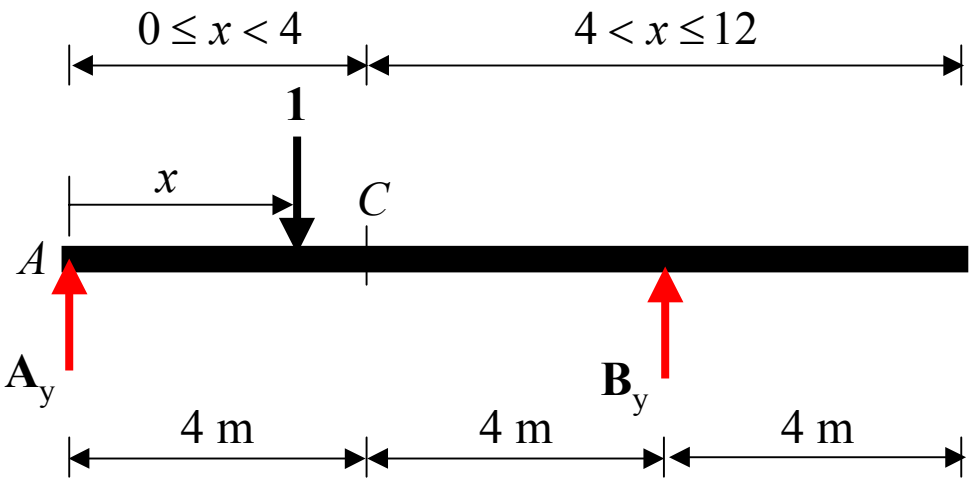
$$V_C = 1 - \frac{1}{8}x$$



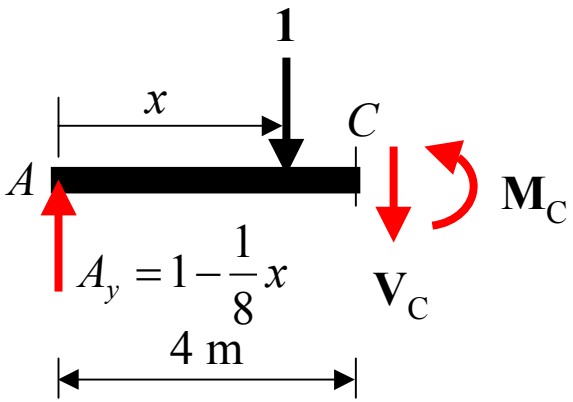
	x	V_C
$V_C = -\frac{1}{8}x$	0	0
	4-	-0.5
$V_C = 1 - \frac{1}{8}x$	4+	0.5
	8	0
	12	-0.5



• **Bending moment at *C***



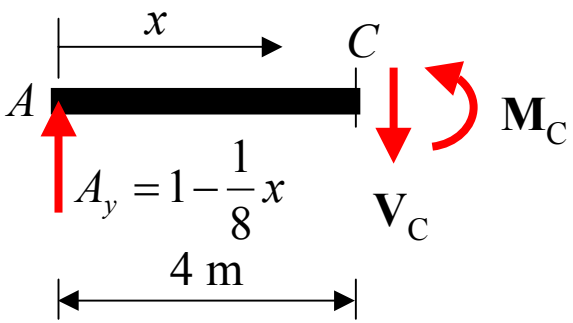
$0 \le x \le 4$



$+\circlearrowleft \Sigma M_C = 0: \quad M_C + 1(4-x) - (1-\frac{1}{8}x)(4) = 0$

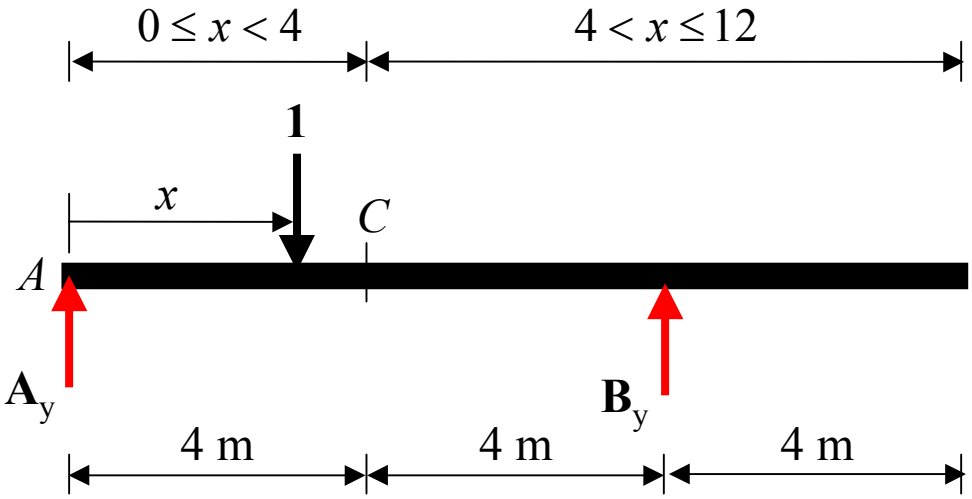
$$M_C = \frac{1}{2}x$$

$4 < x \le 12$

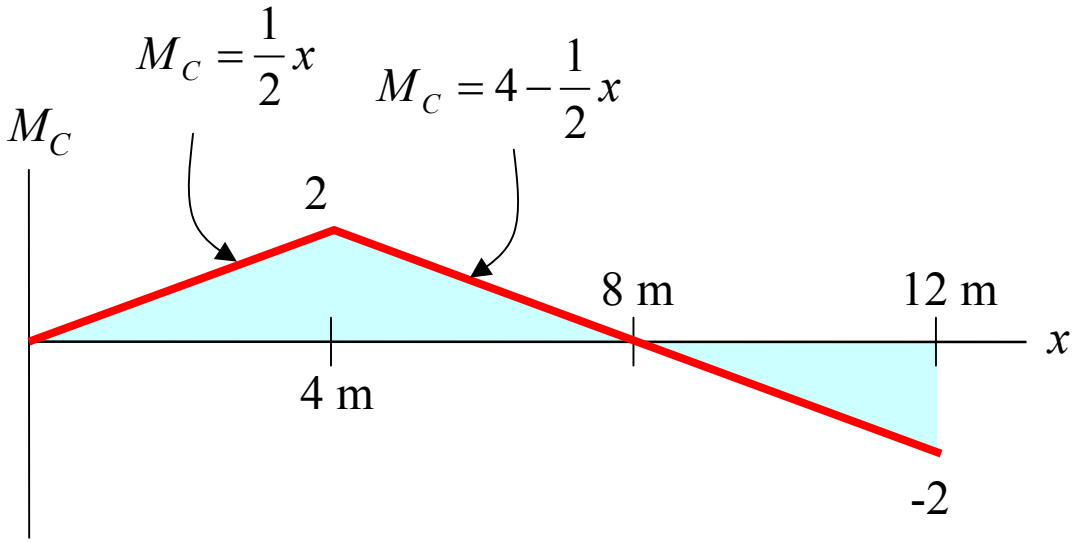


$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (1-\frac{1}{8}x)(4) = 0$

$$M_C = 4 - \frac{1}{2}x$$

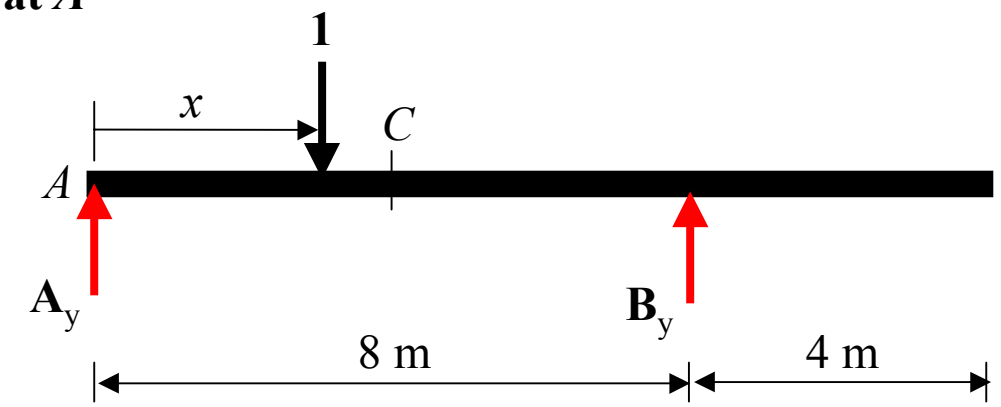


	x	M_C
$M_C = \frac{1}{2}x$	0	0
	4	2
$M_C = 4 - \frac{1}{2}x$	8	0
	12	-2

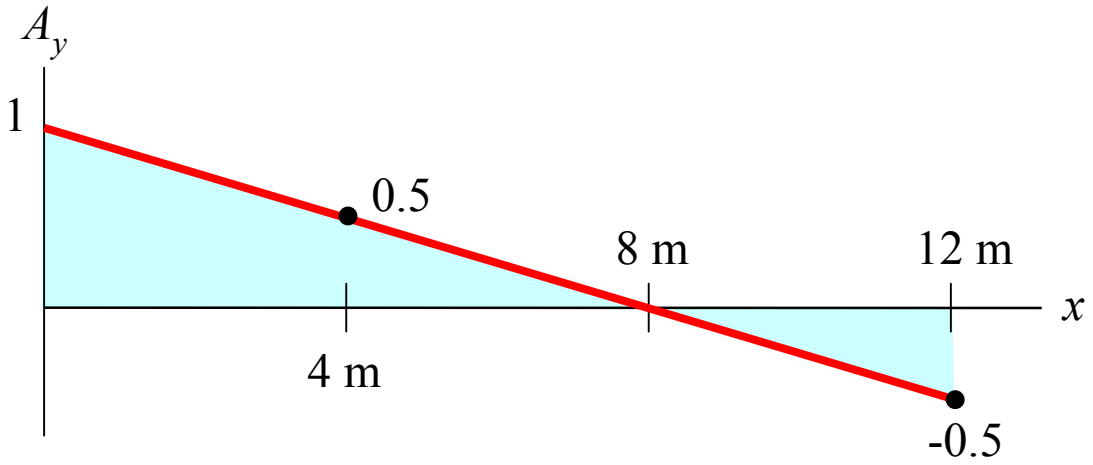


Or using equilibrium conditions:

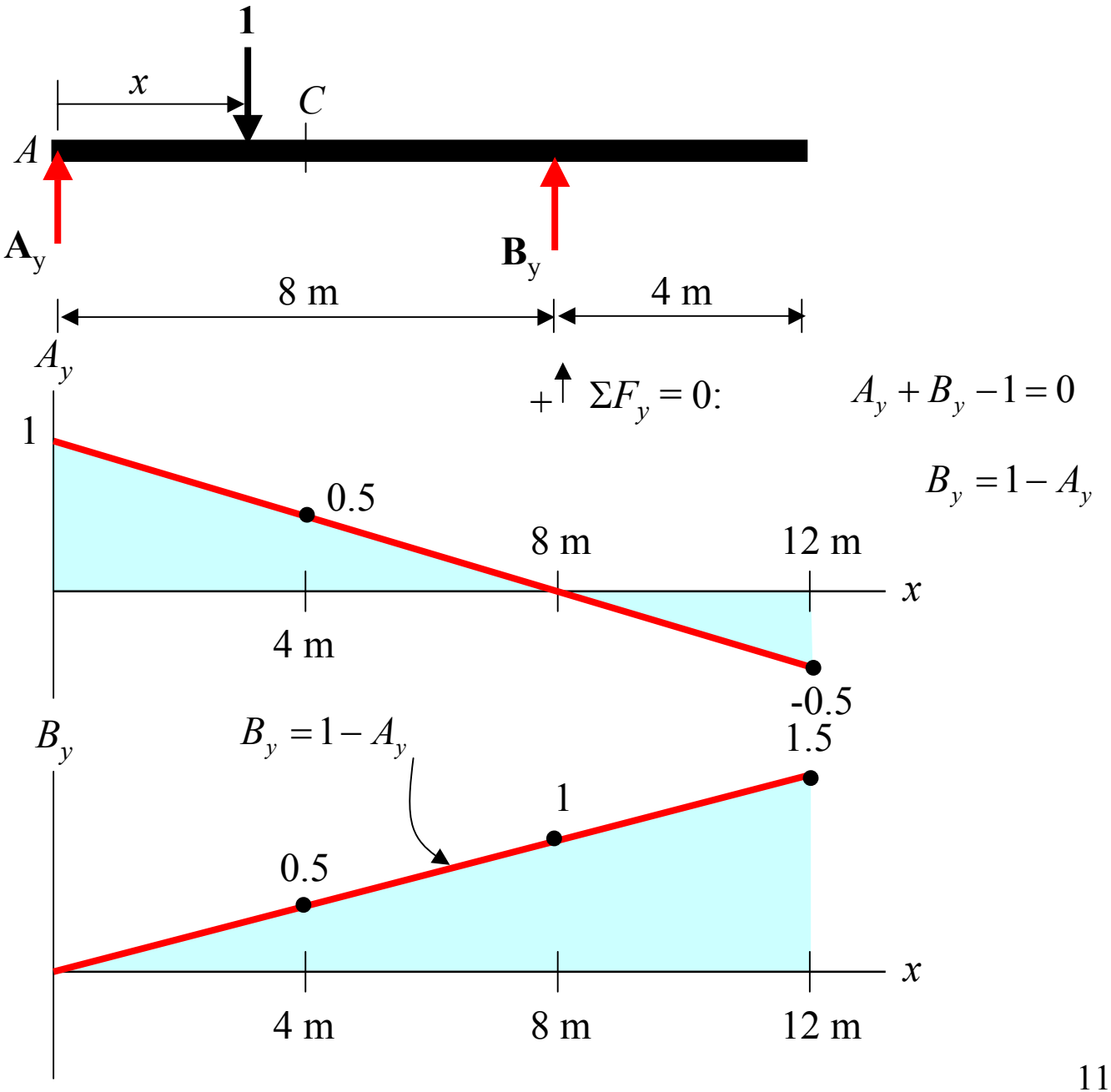
- **Reaction at *A***



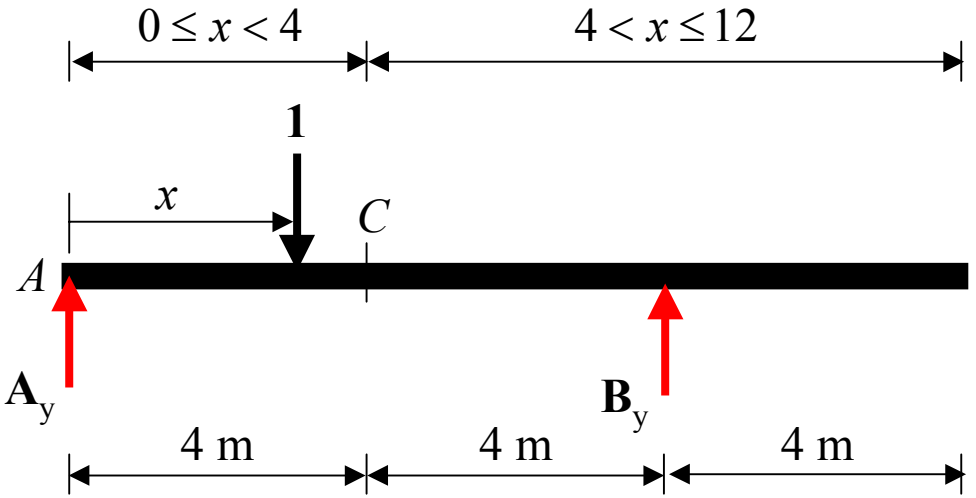
$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(8)+1(8-x)=0, \qquad A_y = 1-\frac{1}{8}x$$



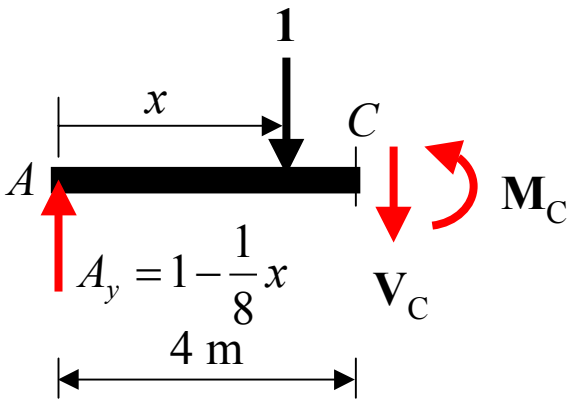
• **Reaction at *B***



• **Shear at *C***



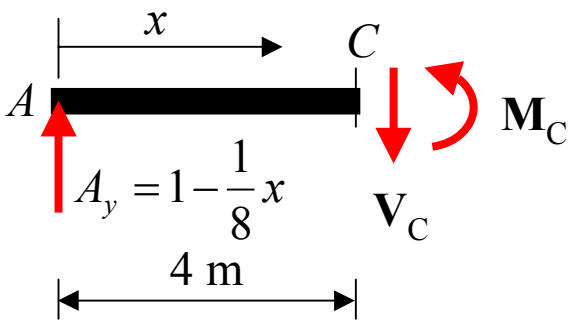
$0 \le x \le 4$



$+\uparrow \Sigma F_y = 0: \qquad A_y - 1 - V_C = 0$

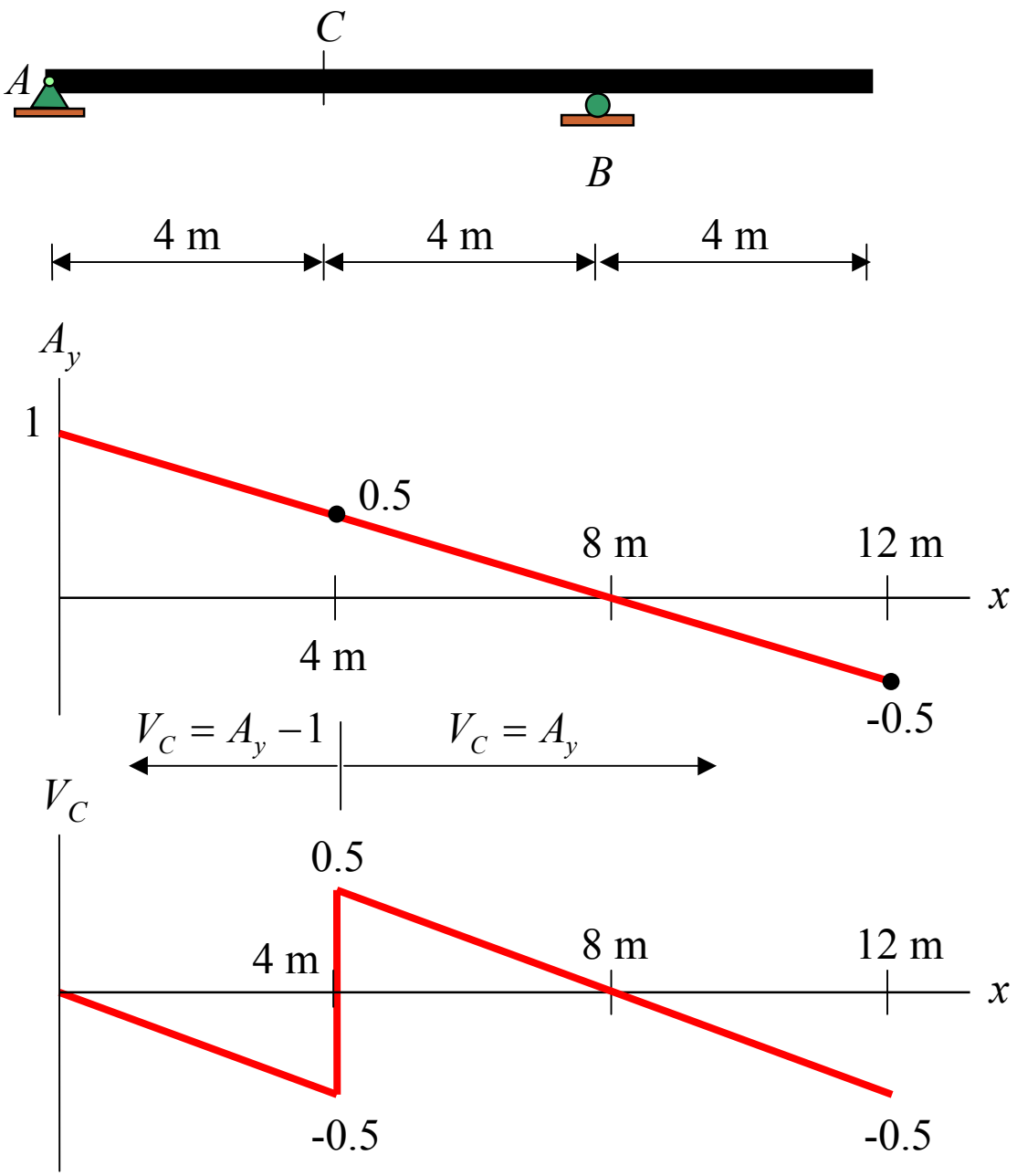
$V_C = A_y - 1$

$4 < x \le 12$

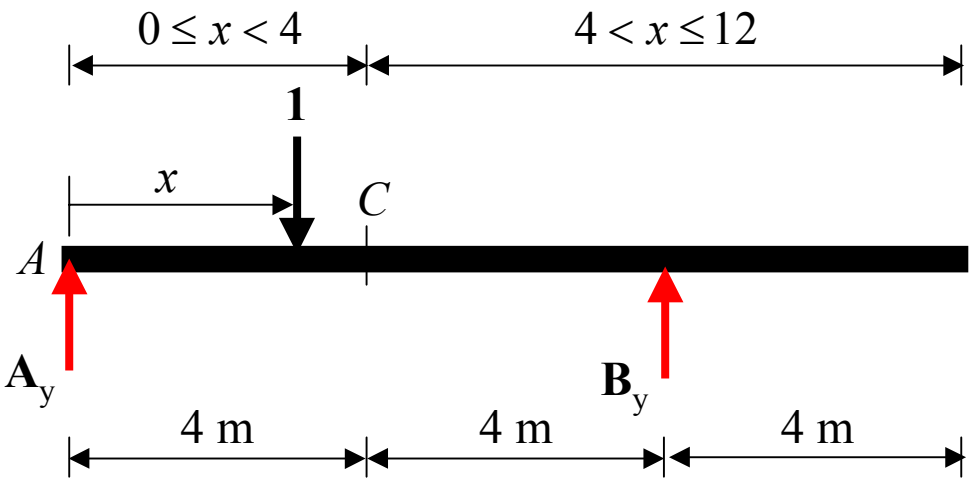


$+\uparrow \Sigma F_y = 0: \qquad A_y - V_C = 0$

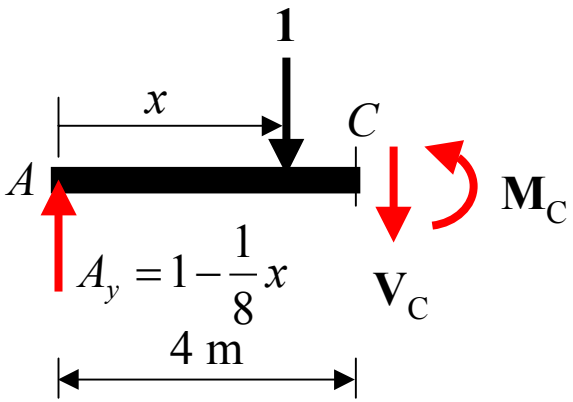
$V_C = A_y$


C
A
B
4 m
4 m
4 m
A_y
1
0.5
8 m
12 m
x
4 m
-0.5
$V_C = A_y - 1$
$V_C = A_y$
V_C
0.5
4 m
8 m
12 m
x
-0.5
-0.5

• **Bending moment at *C***



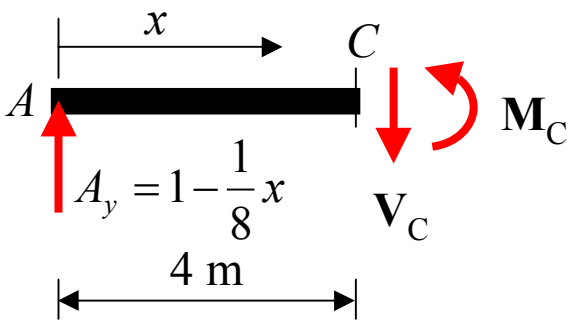
$0 \le x \le 4$



$+\circlearrowleft \Sigma M_C = 0:$ $\quad A_y(4) + 1(4-x) + M_C = 0$

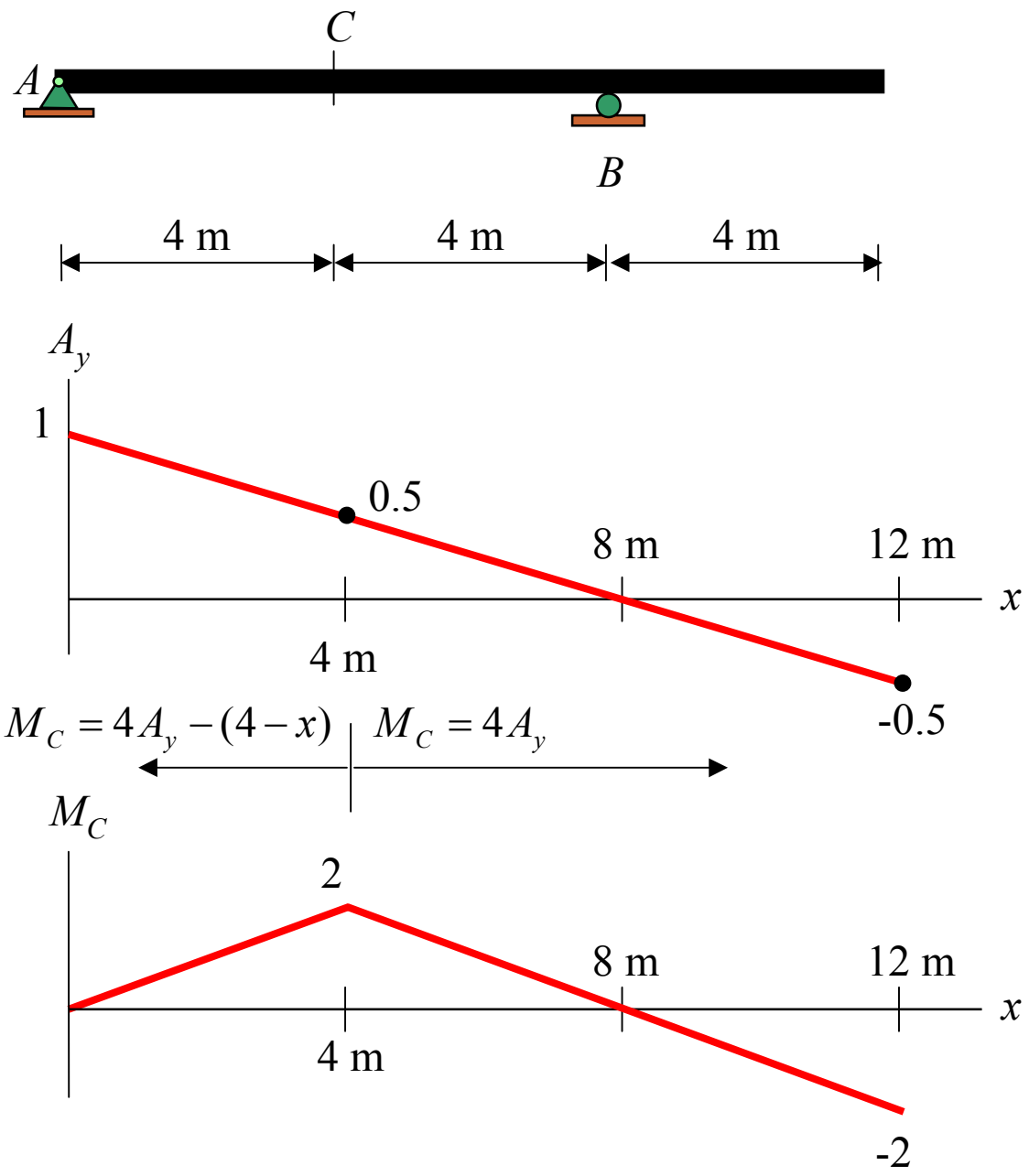
$M_C = 4A_y - (4-x)$

$4 < x \le 12$

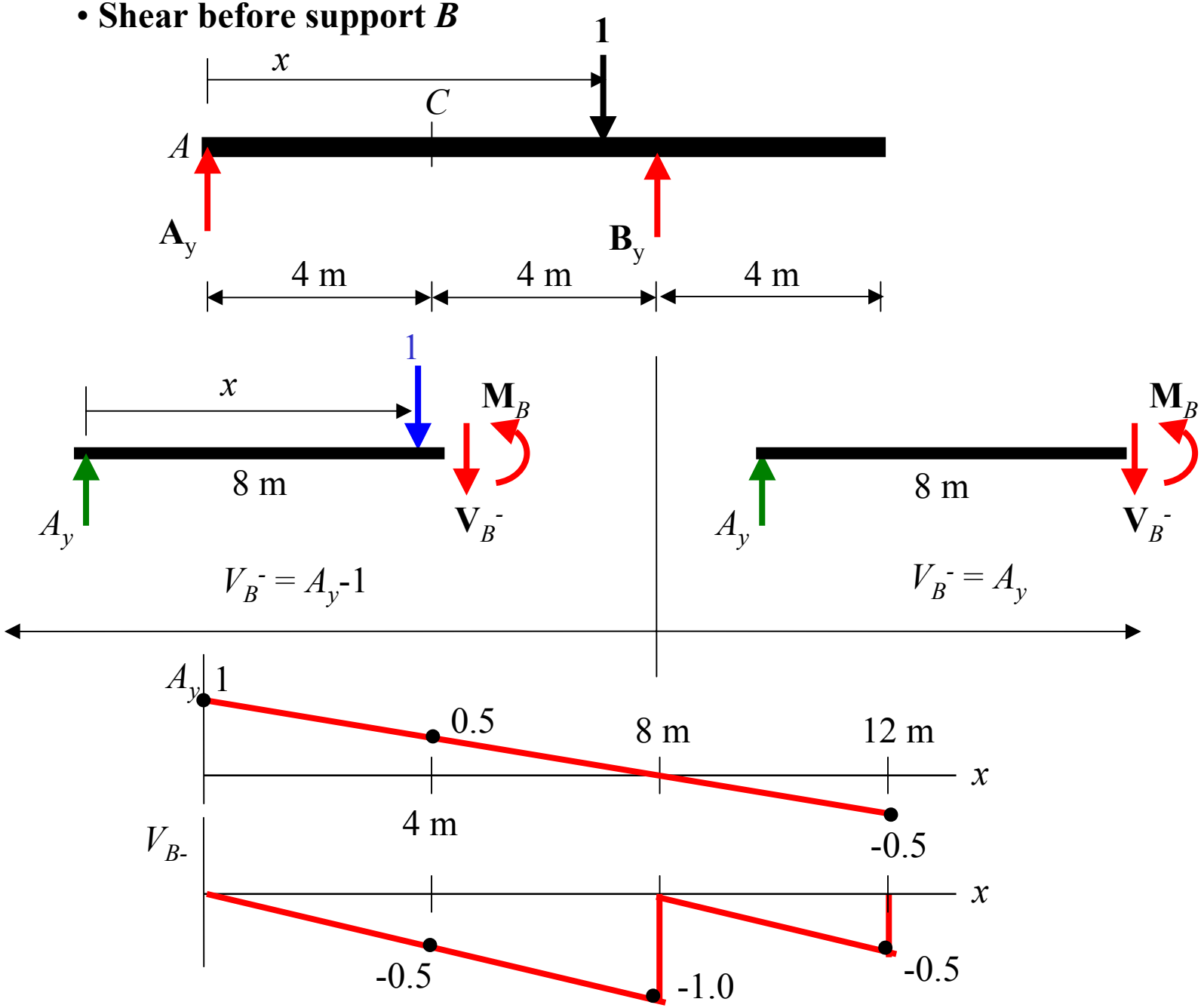


$+\circlearrowleft \Sigma M_C = 0:$ $\quad -A_y(4) + M_C = 0$

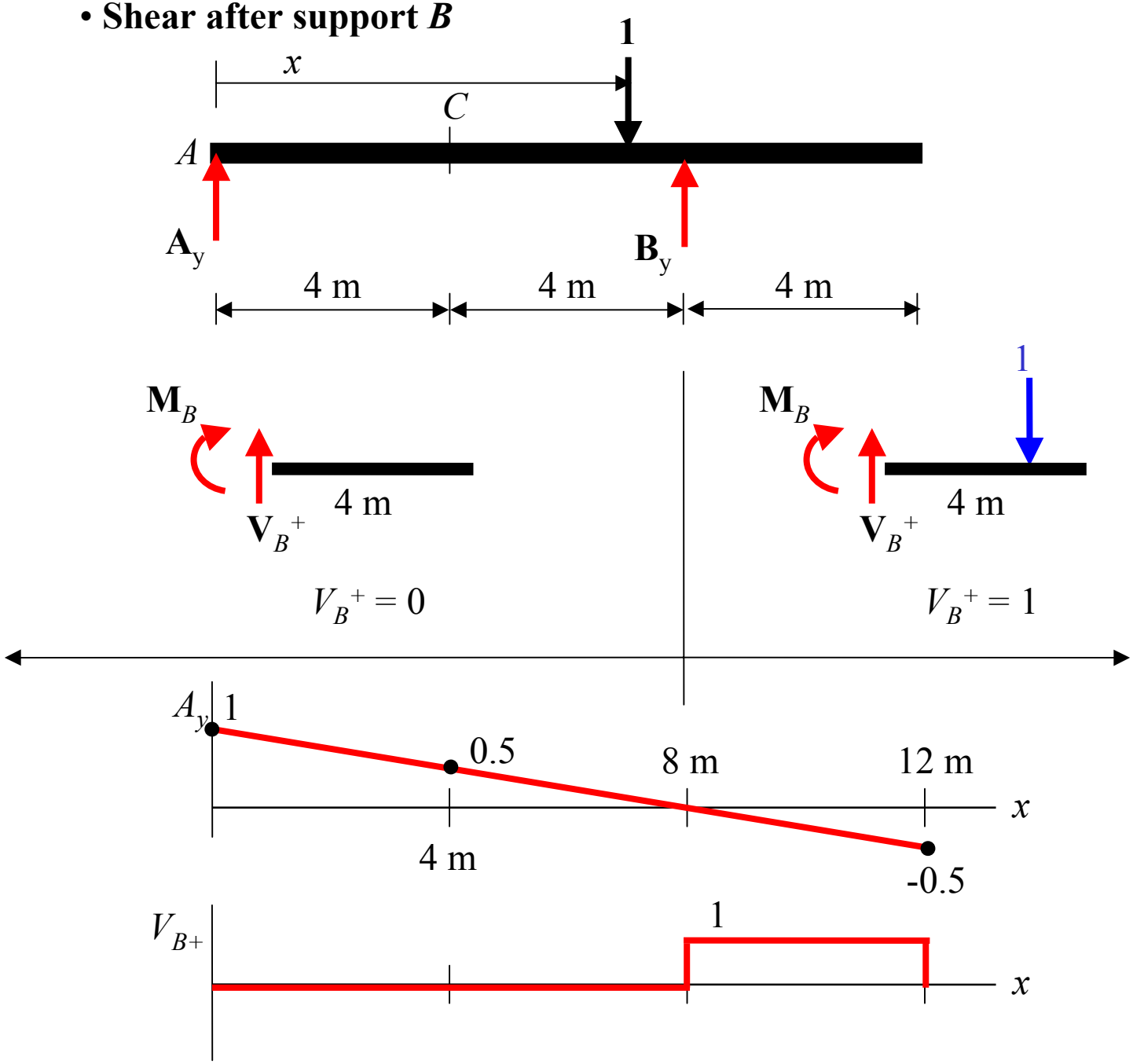
$M_C = 4A_y$


C
A
B
4 m
4 m
4 m
A_y
1
0.5
8 m
12 m
x
4 m
-0.5
$M_C = 4A_y - (4 - x)$
$M_C = 4A_y$
M_C
2
8 m
12 m
x
4 m
-2

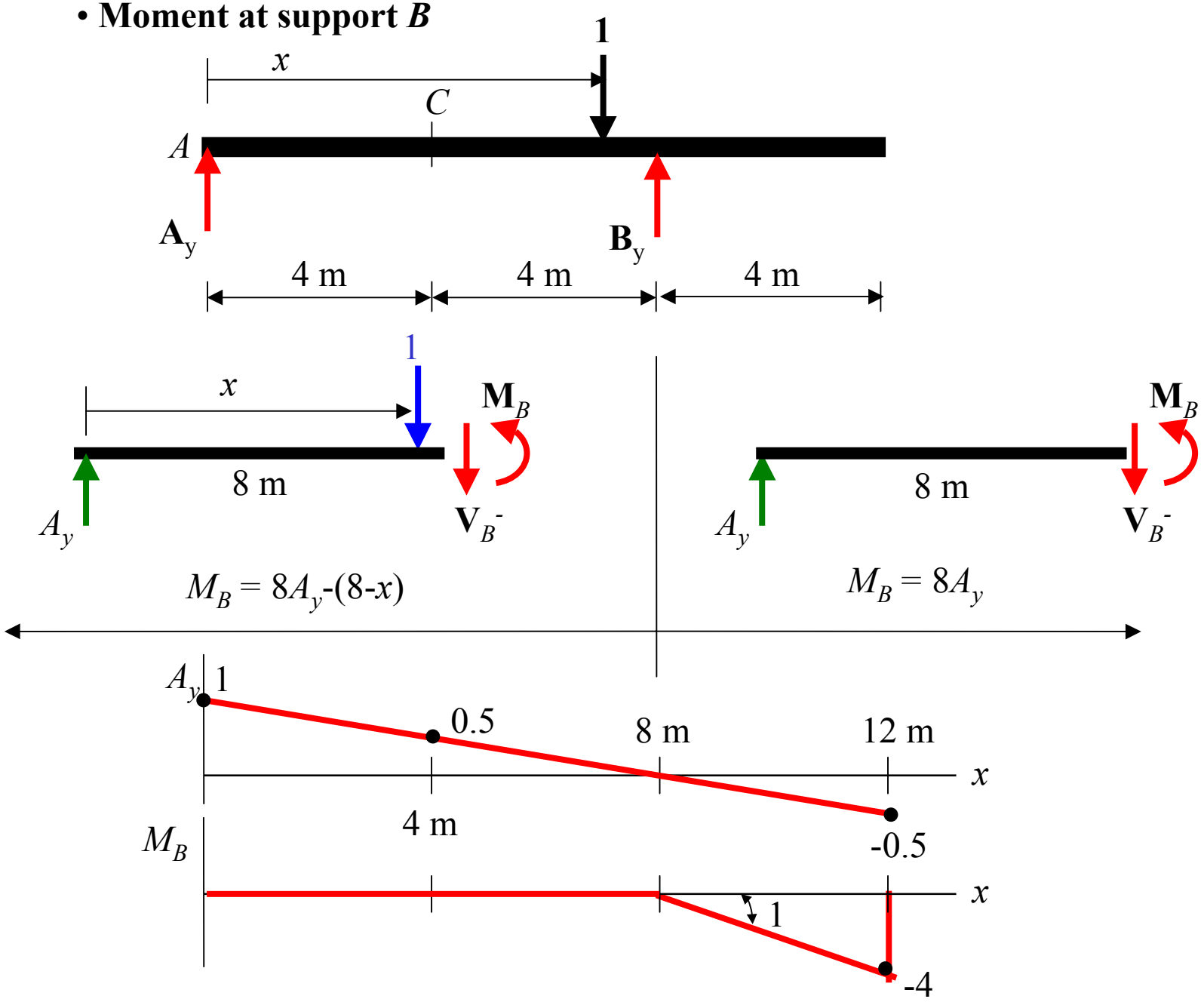
• **Shear before support *B***



• **Shear after support *B***

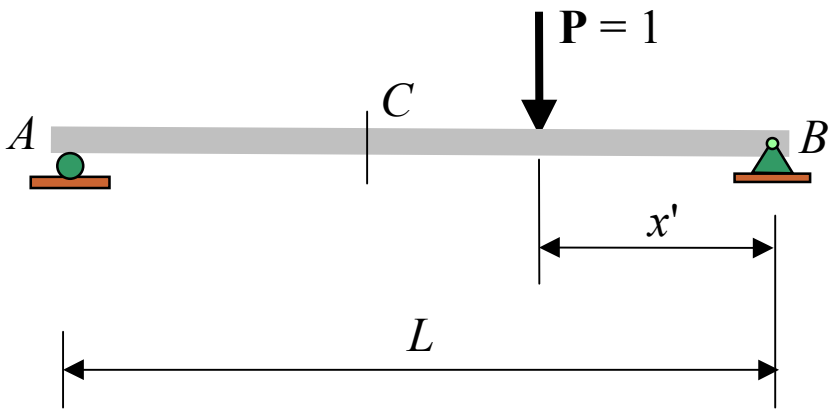


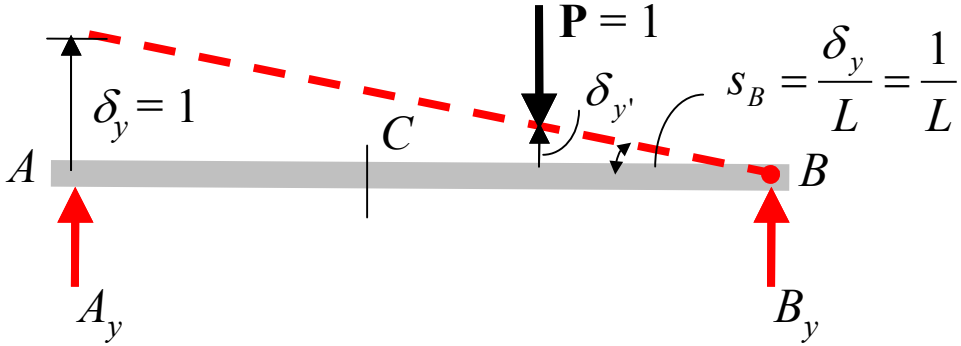
• **Moment at support *B***



Influence Line for Beam

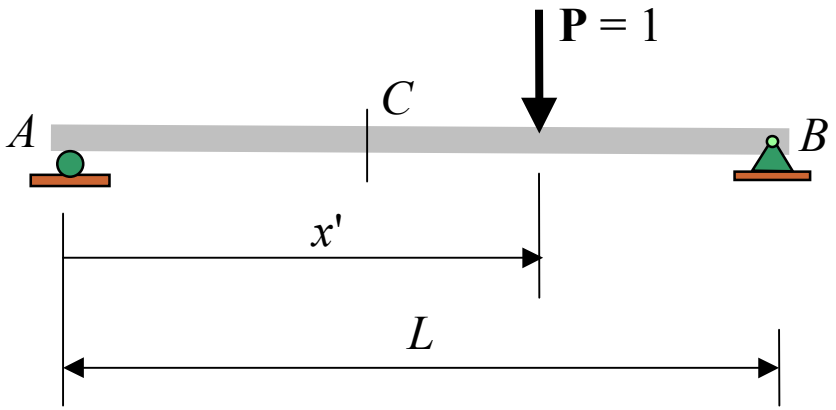
- **Reaction**

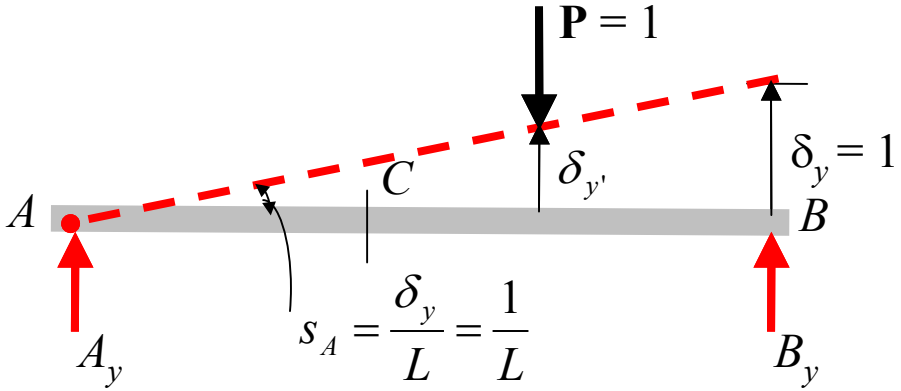




$$A_y(1) - 1(\delta_{y'}) + B_y(0) = 0$$

$$A_y = \delta_{y'}$$

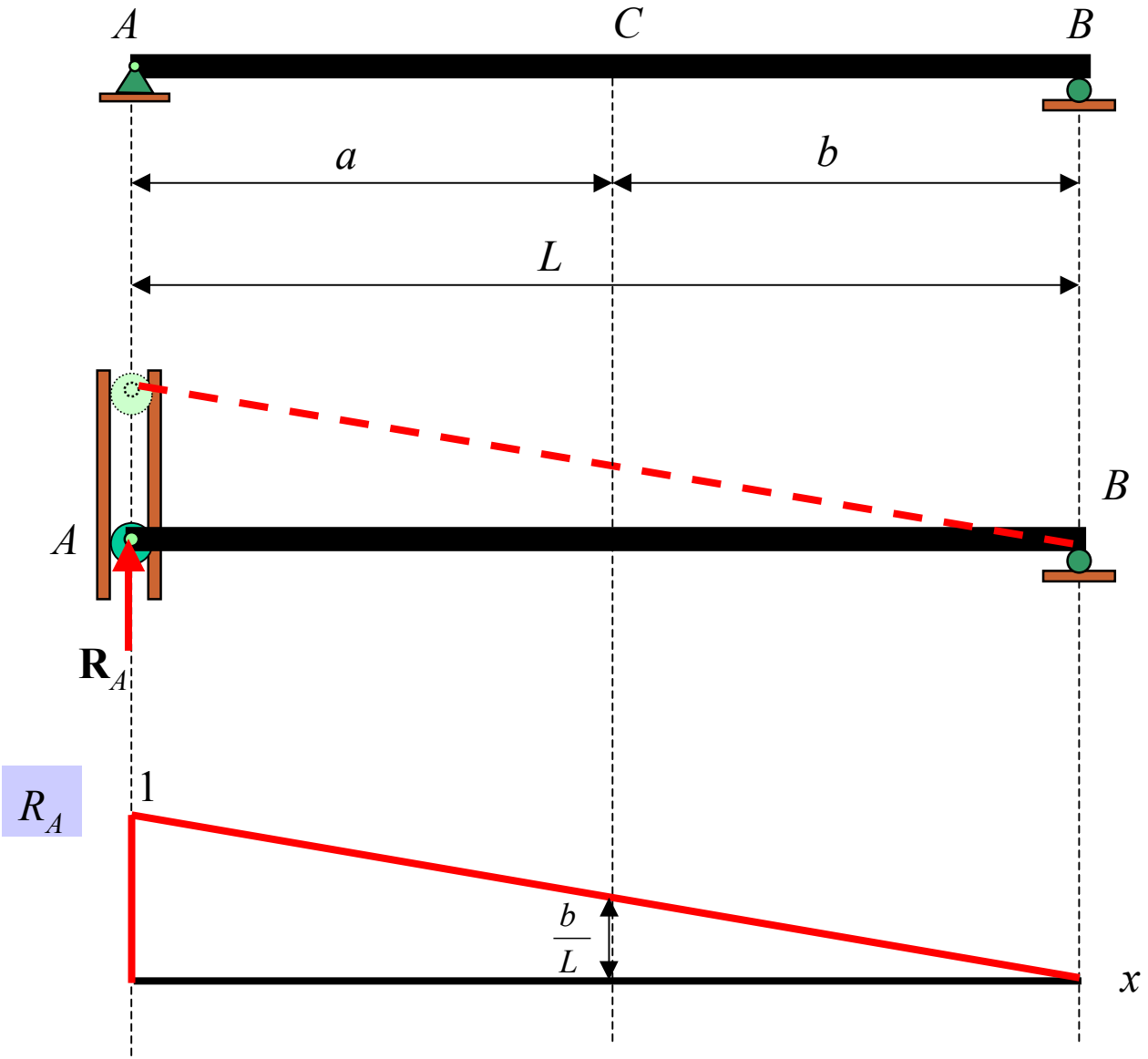




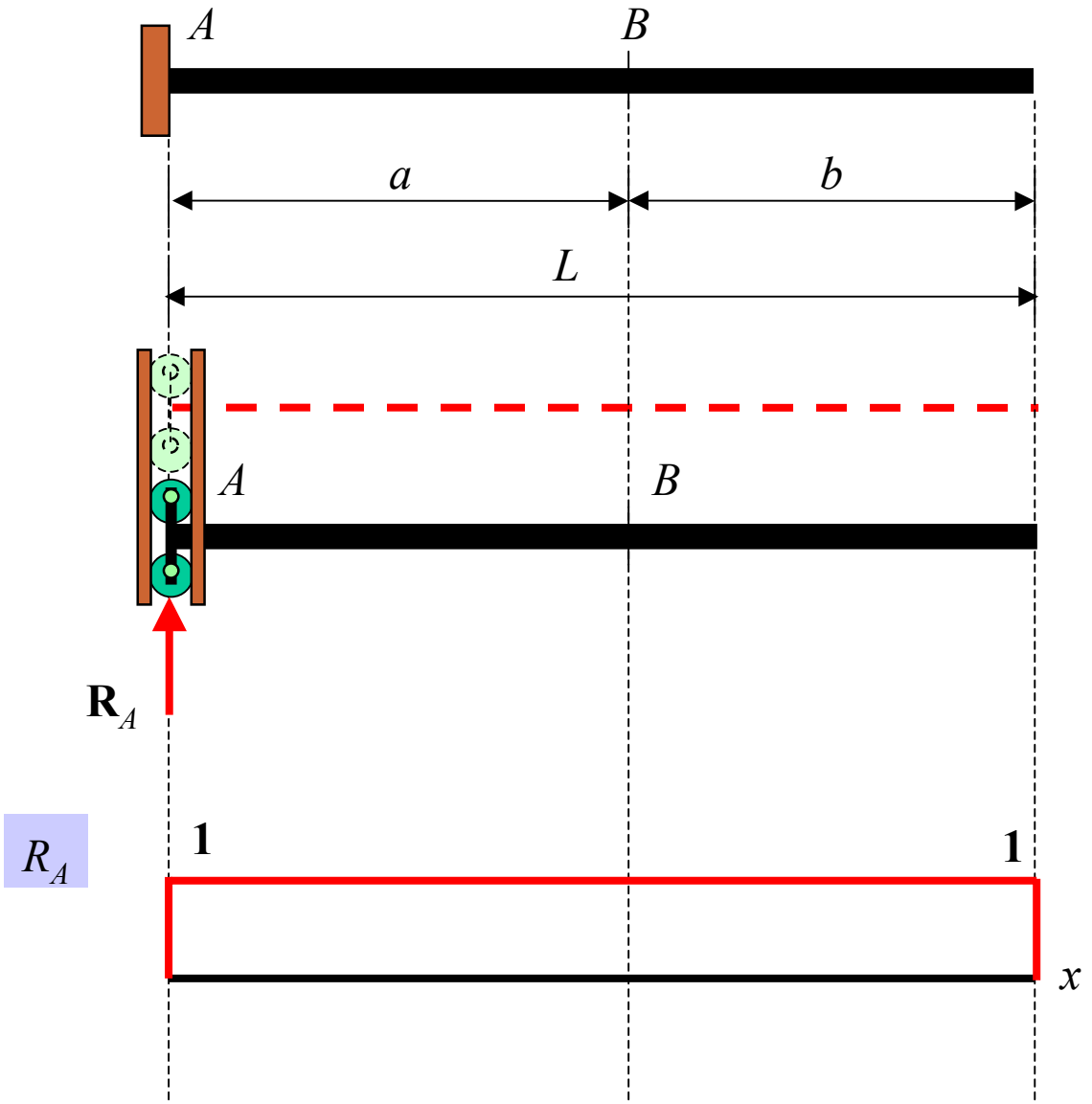
$$A_y(0) - 1(\delta_{y'}) + B_y(1) = 0$$

$$B_y = \delta_{y'}$$

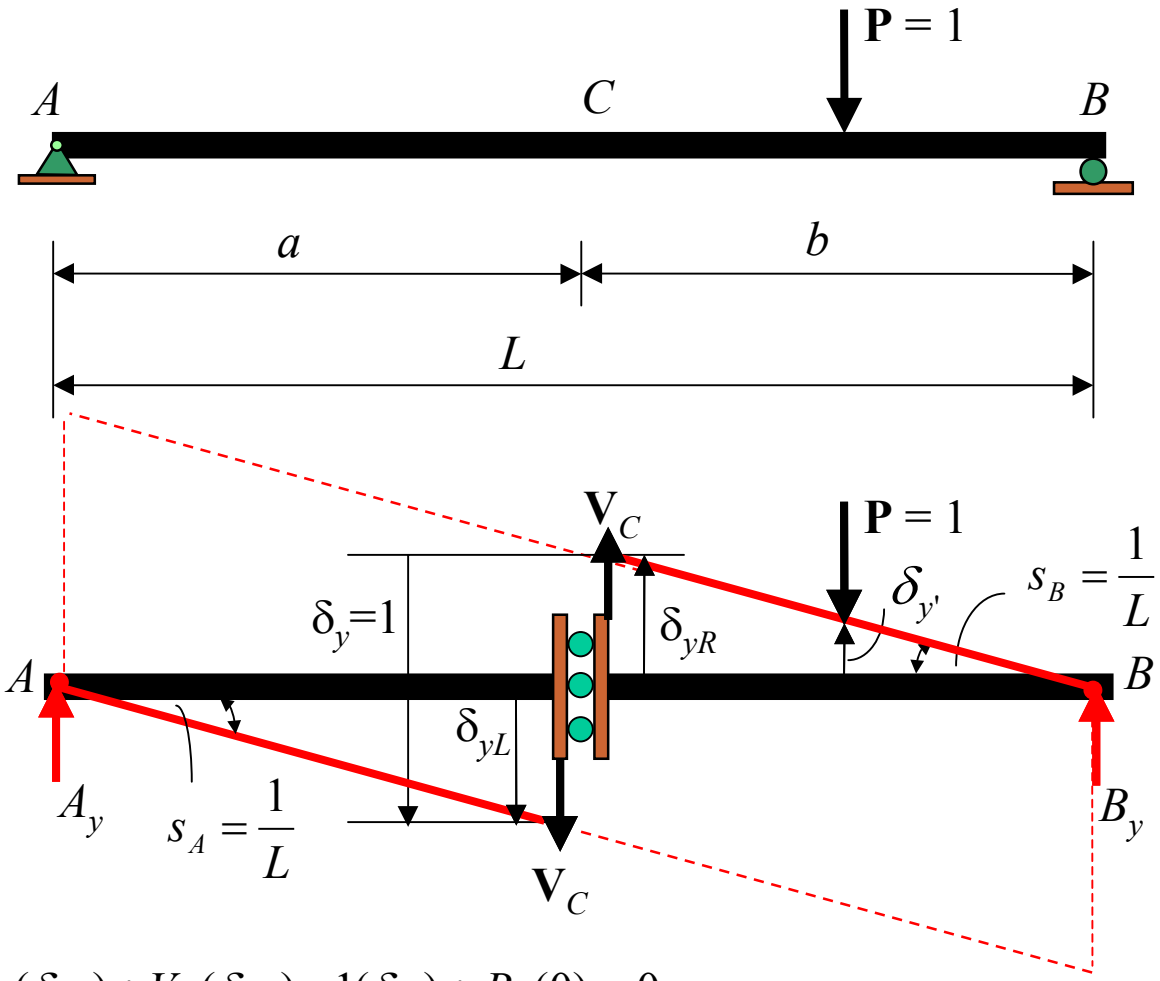
- Pinned Support



- Fixed Support



• **Shear**



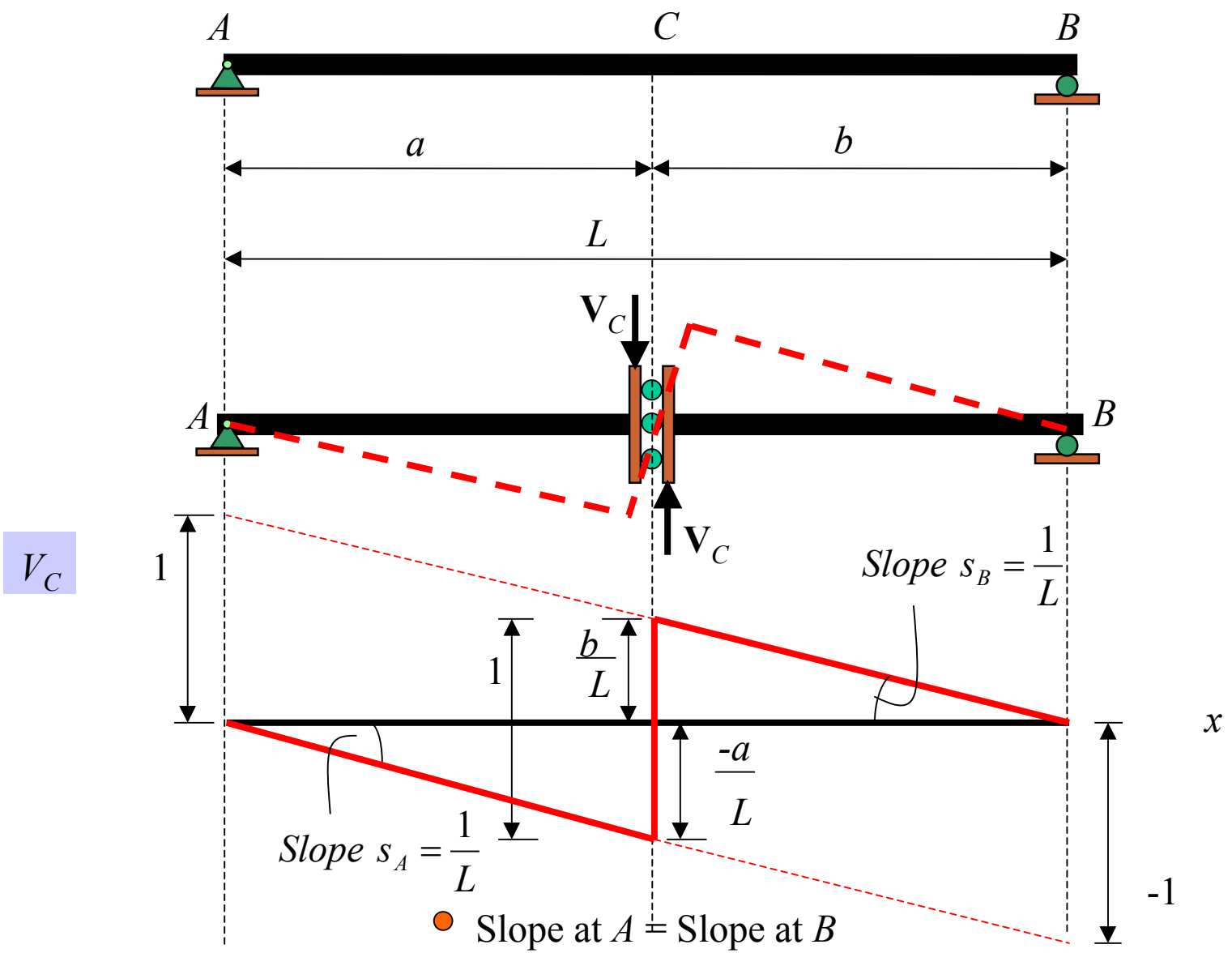
$$A_y(0) + V_C(\delta_{yL}) + V_C(\delta_{yR}) - 1(\delta_{y'}) + B_y(0) = 0$$

$$V_C\underbrace{(\delta_{yL} + \delta_{yR})}_{\delta_y=1} = \delta_{y'}$$

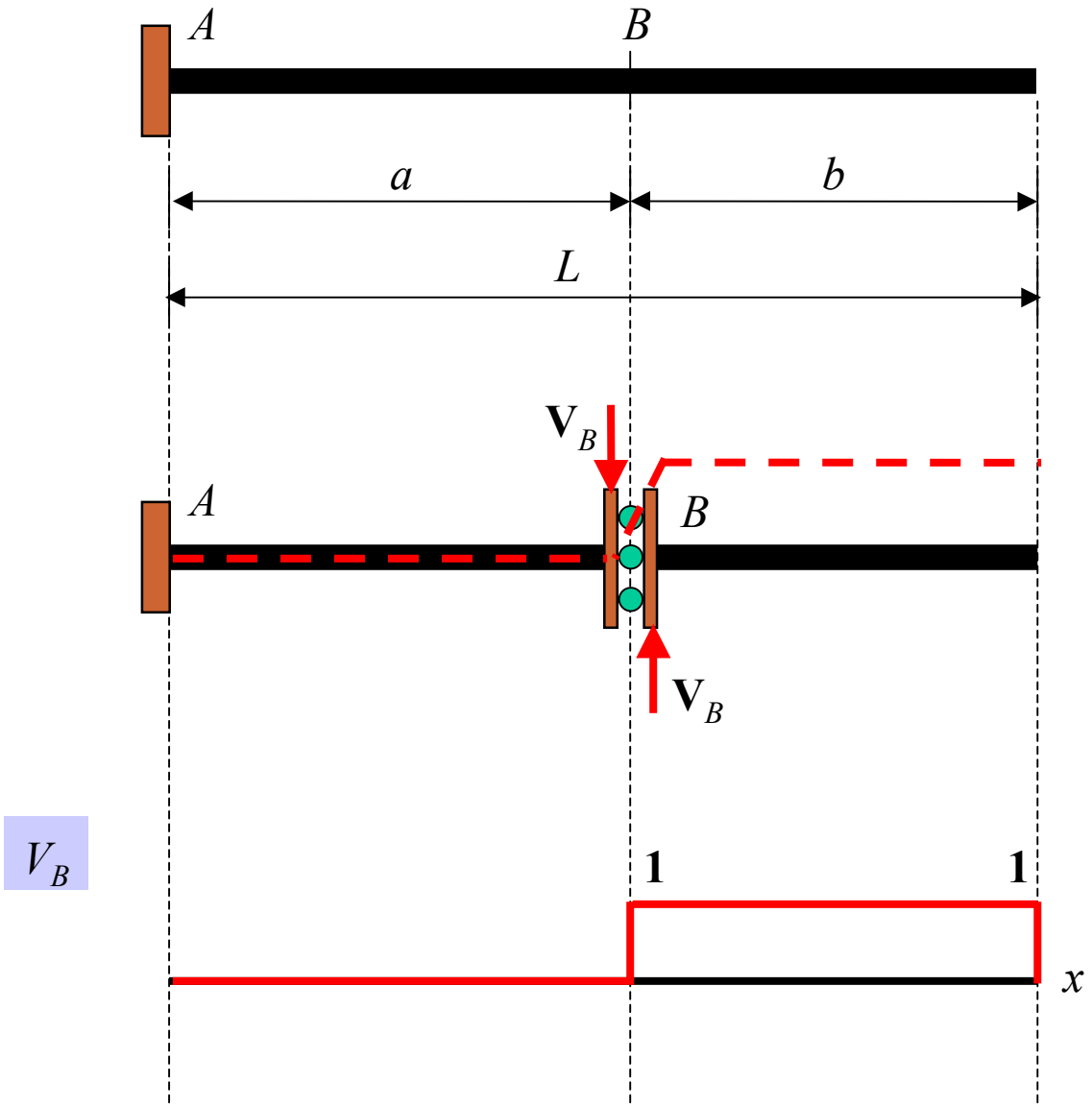
$$V_C = \delta_{y'}$$

$$slopes: \quad |s_A| = |s_B|$$

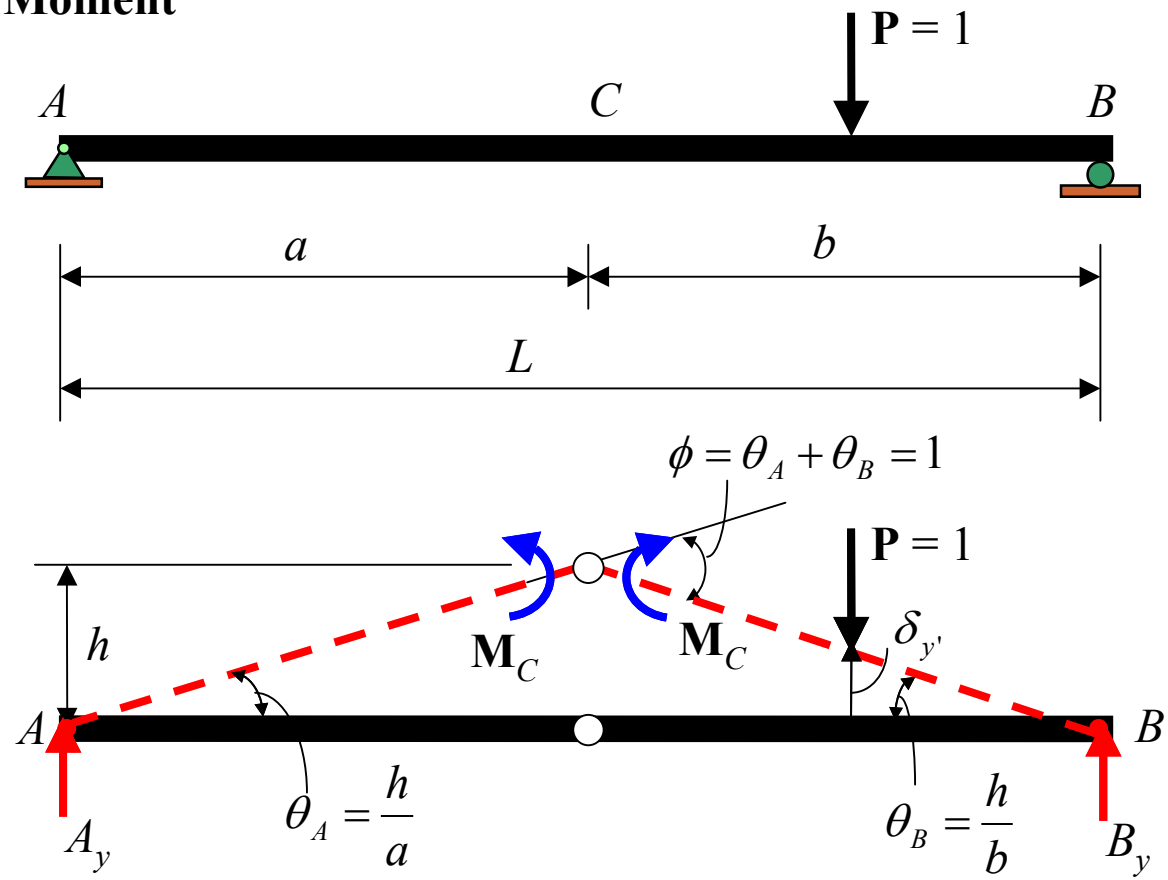
- Pinned Support



- Fixed Support



• **Bending Moment**



$$A_y(0) + M_C(\theta_A) + M_C(\theta_B) + 1(\delta_{y'}) + B_y(0) = 0$$

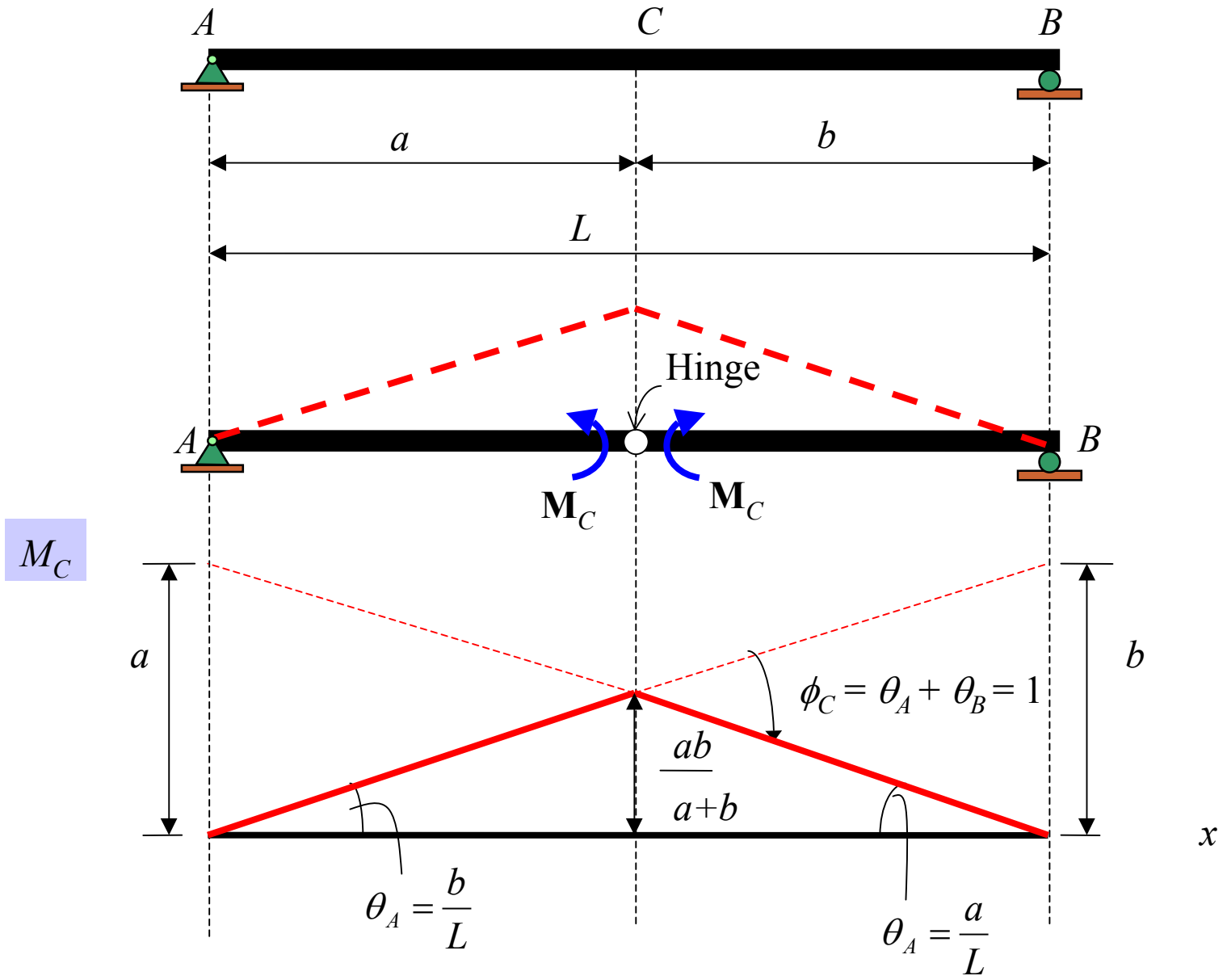
$$M_C(\theta_A + \theta_B) = \delta_{y'} \quad (1)$$

$$M_C = \delta_{y'}$$

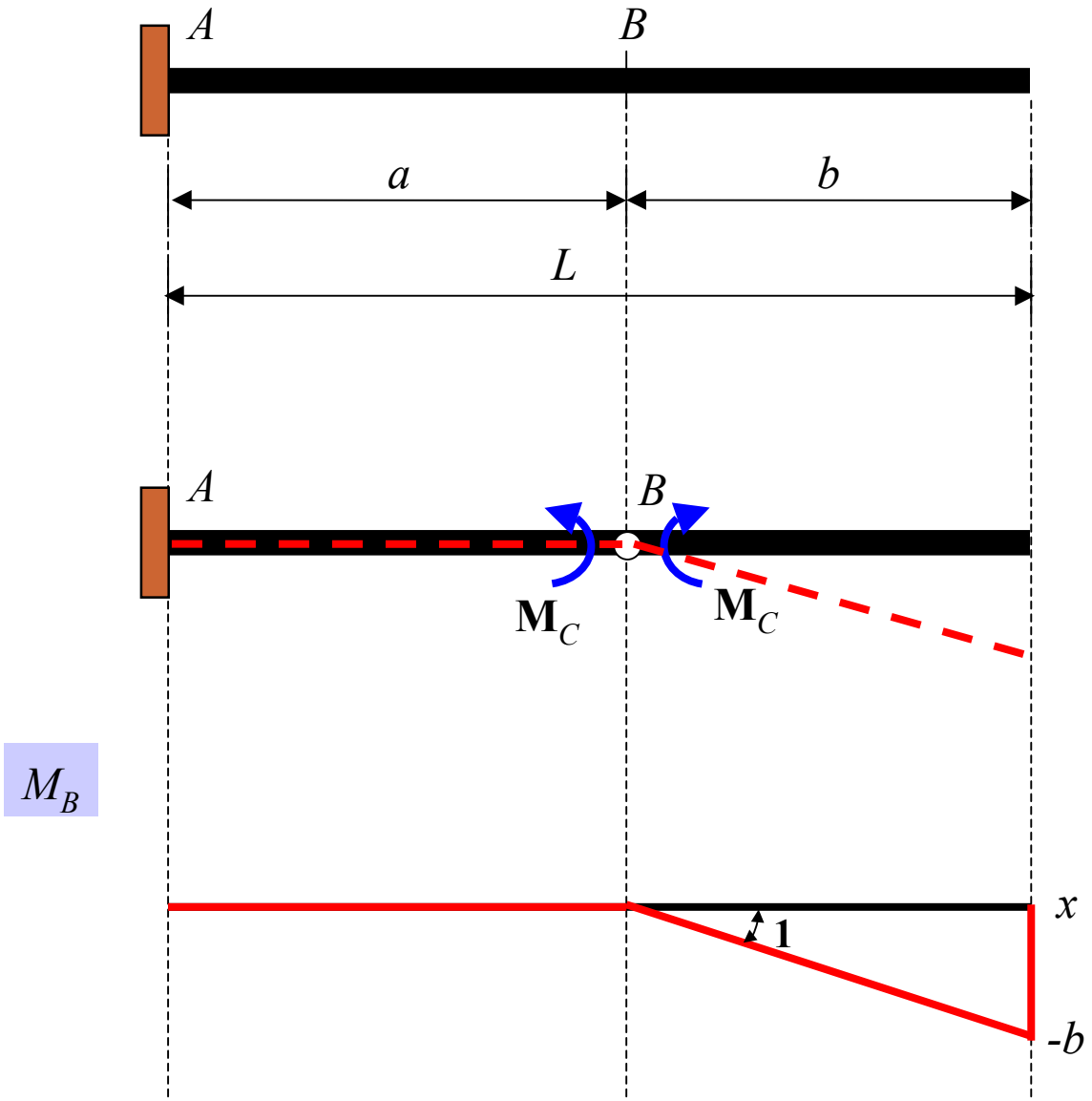
$$\left(\frac{h}{a} + \frac{h}{b}\right) = 1$$

$$\frac{h(a+b)}{ab} = 1, \qquad h = \frac{ab}{(a+b)}$$

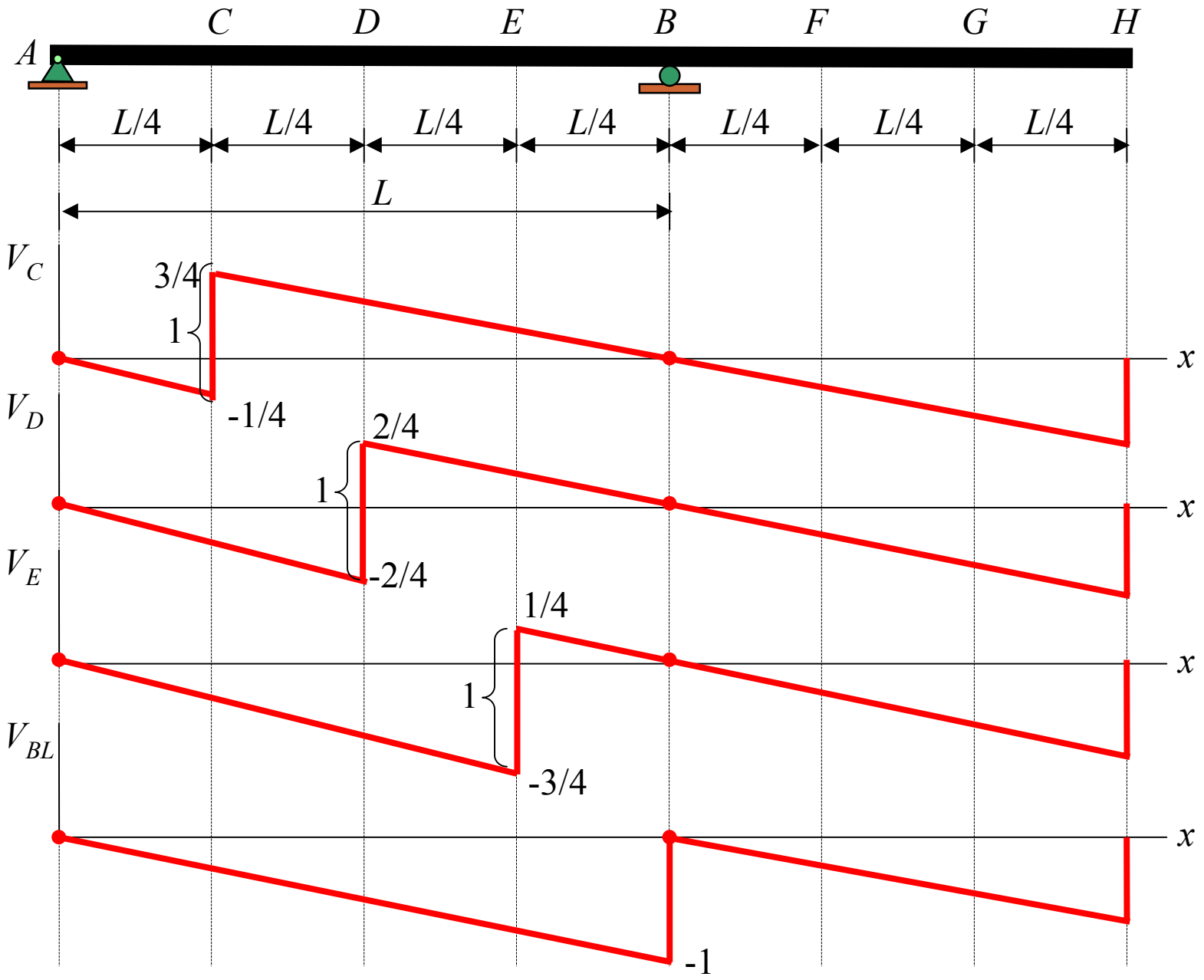
- Pinned Support

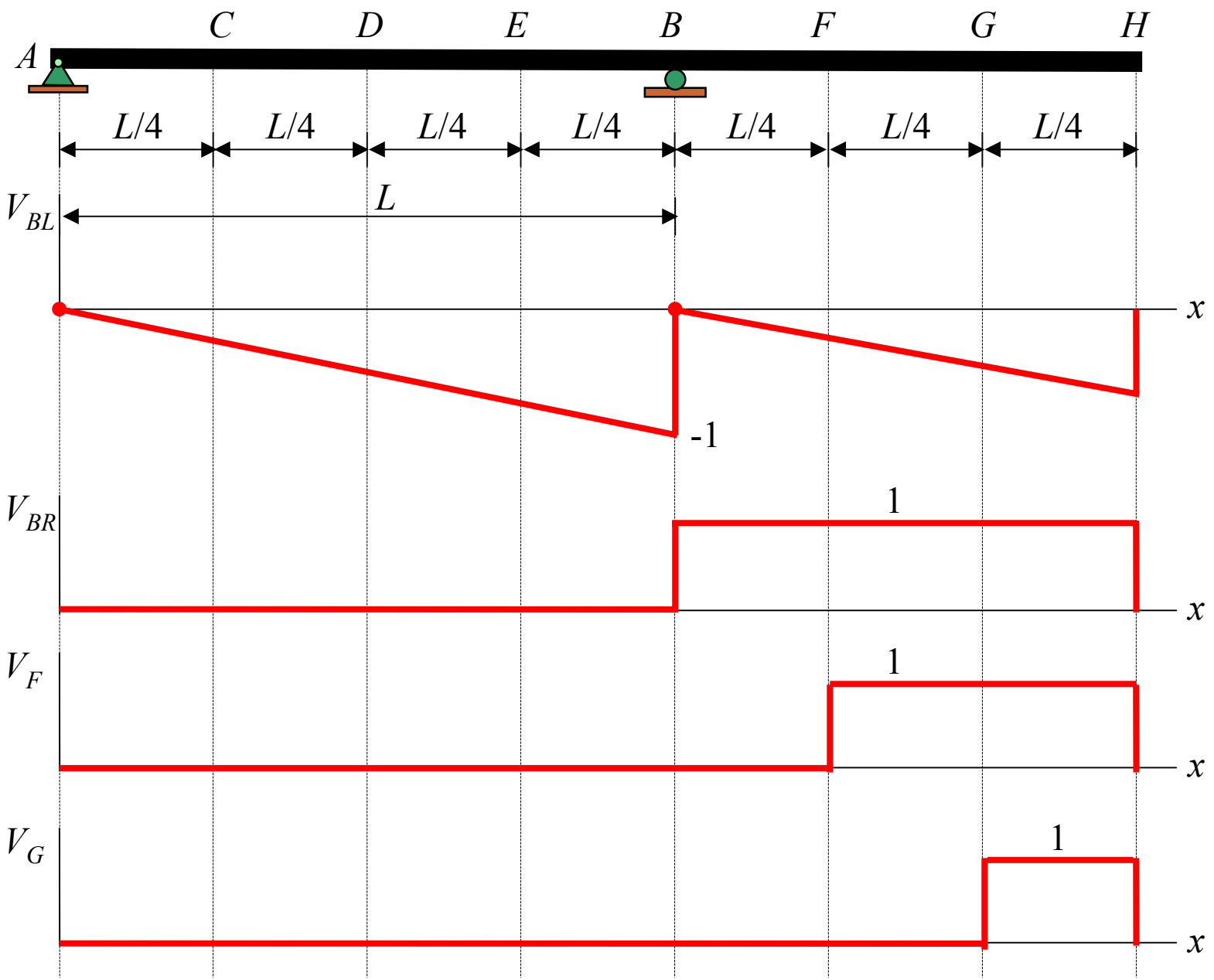


- Fixed Support

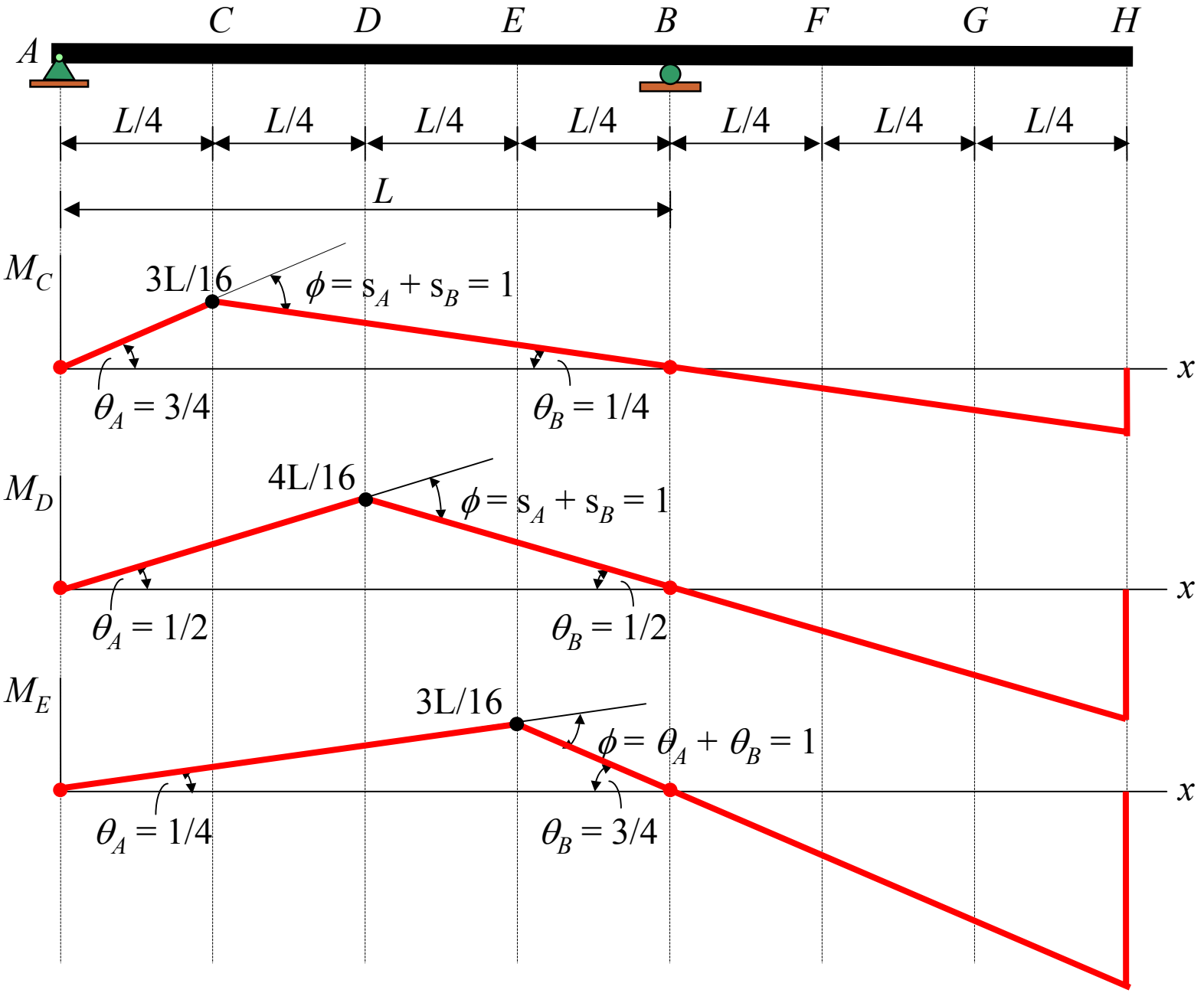


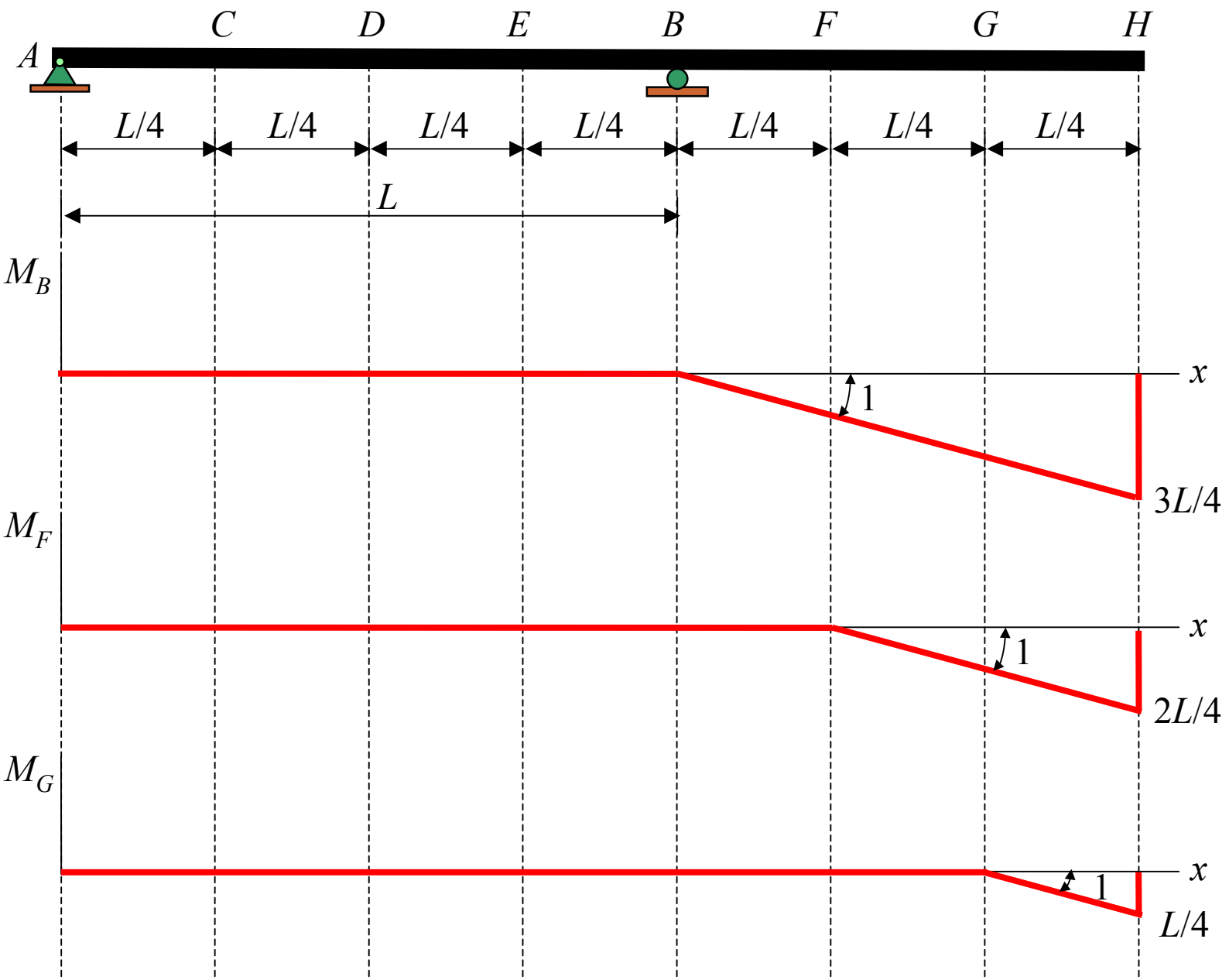
• **General Shear**




A
C
D
E
B
F
G
H
L/4
L/4
L/4
L/4
L/4
L/4
L/4
V_{BL}
L
x
-1
V_{BR}
1
x
V_F
1
x
V_G
1
x

• **General Bending Moment**

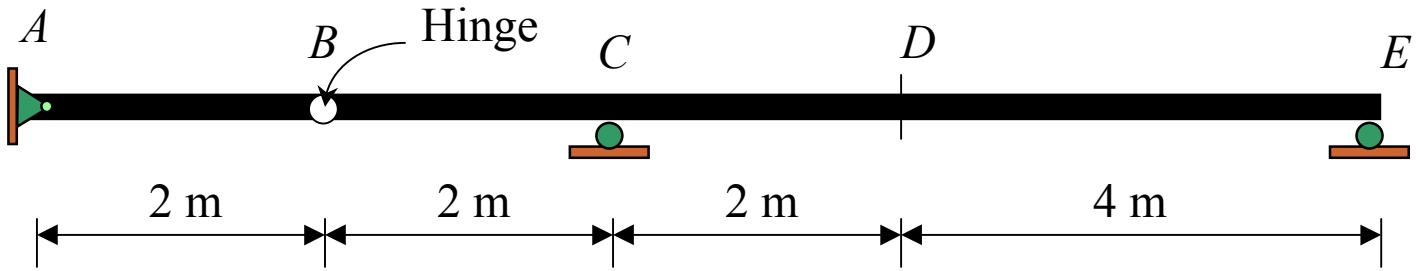



A
C
D
E
B
F
G
H
L/4
L/4
L/4
L/4
L/4
L/4
L/4
L
M_B
x
1
3L/4
M_F
x
1
2L/4
M_G
x
1
L/4

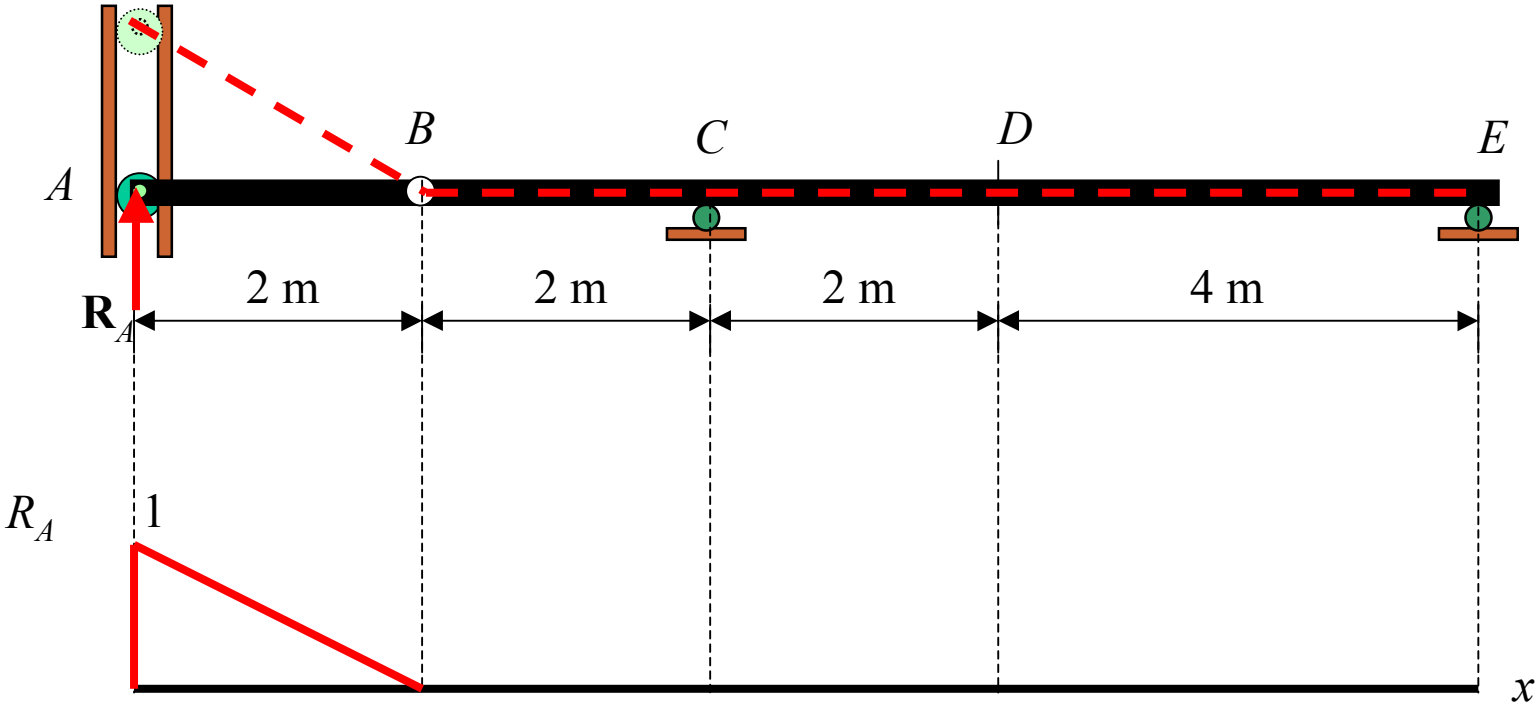
Example 6-2

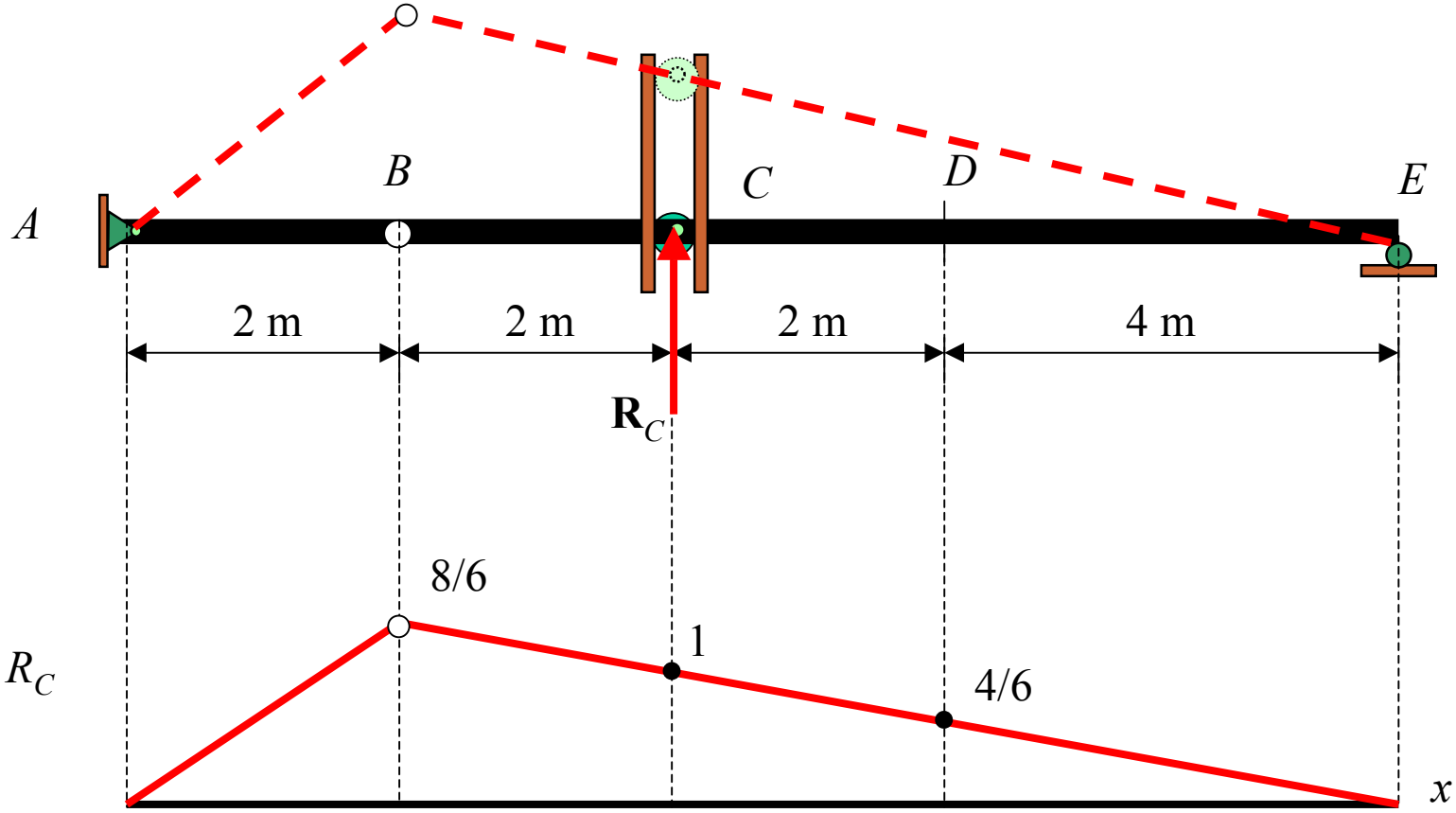
Construct the influence line for

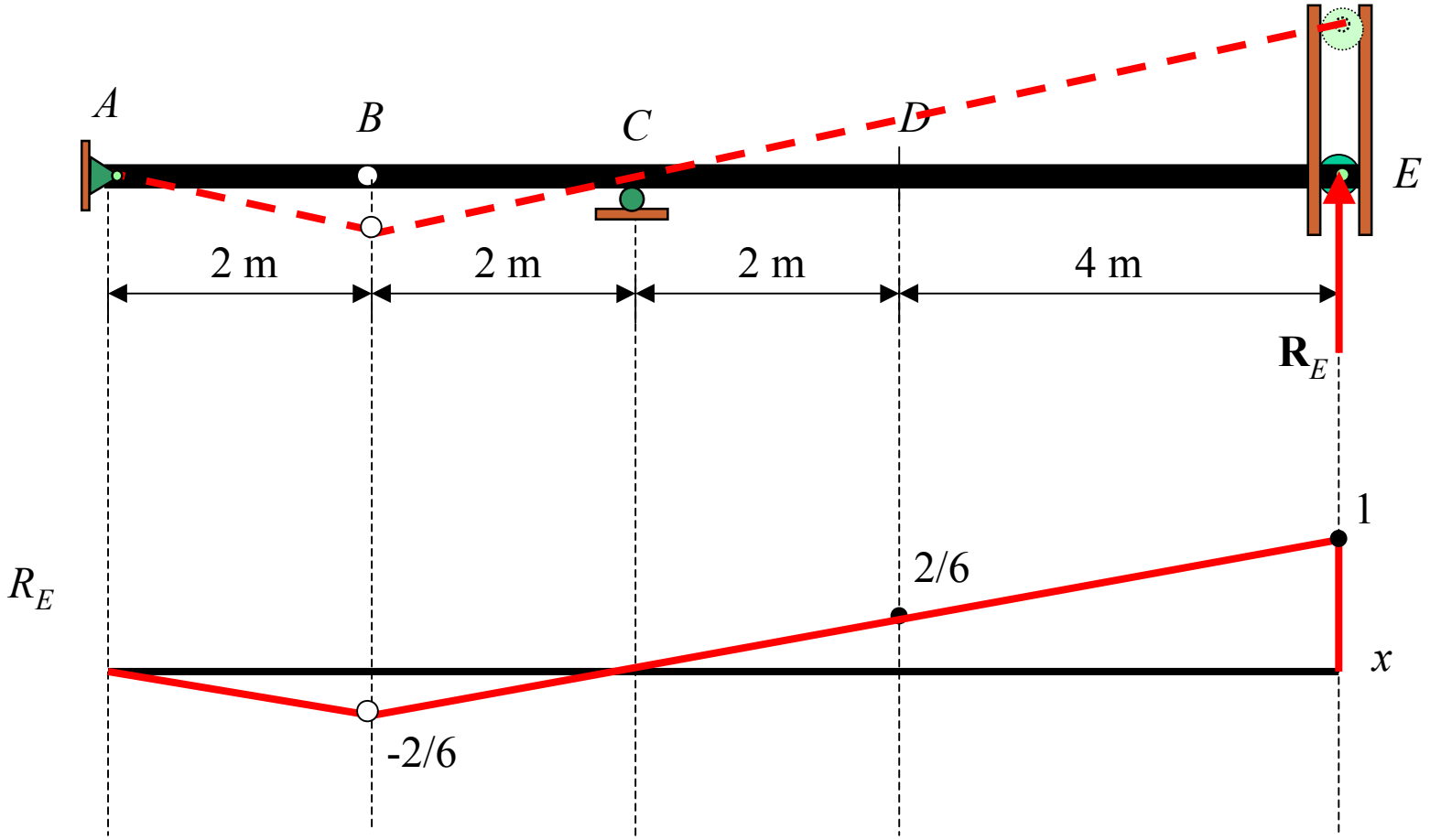
- the reaction at *A, C* and *E*
- the shear at *D*
- the moment at *D*
- shear before and after support *C*
- moment at point *C*

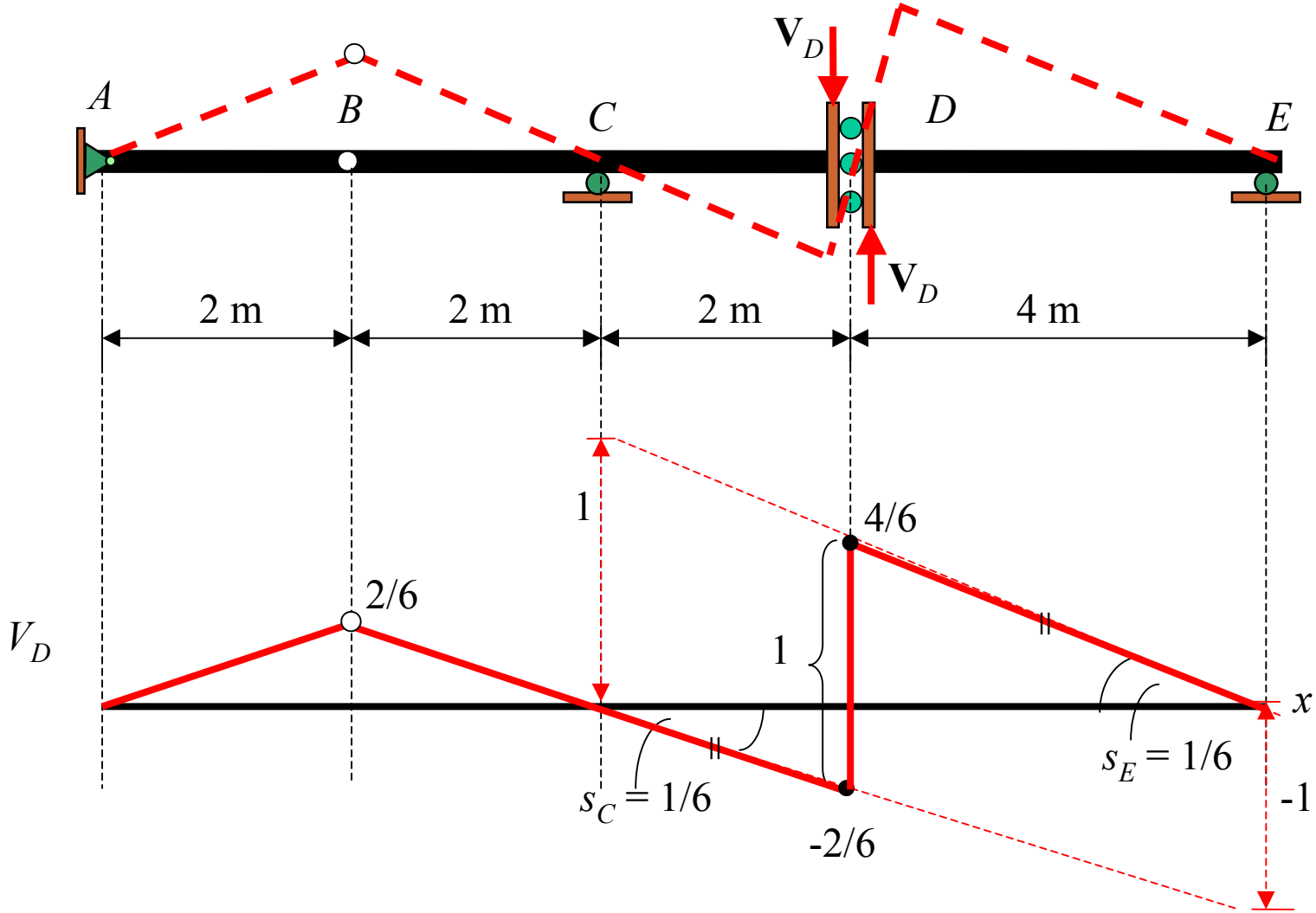


SOLUTION



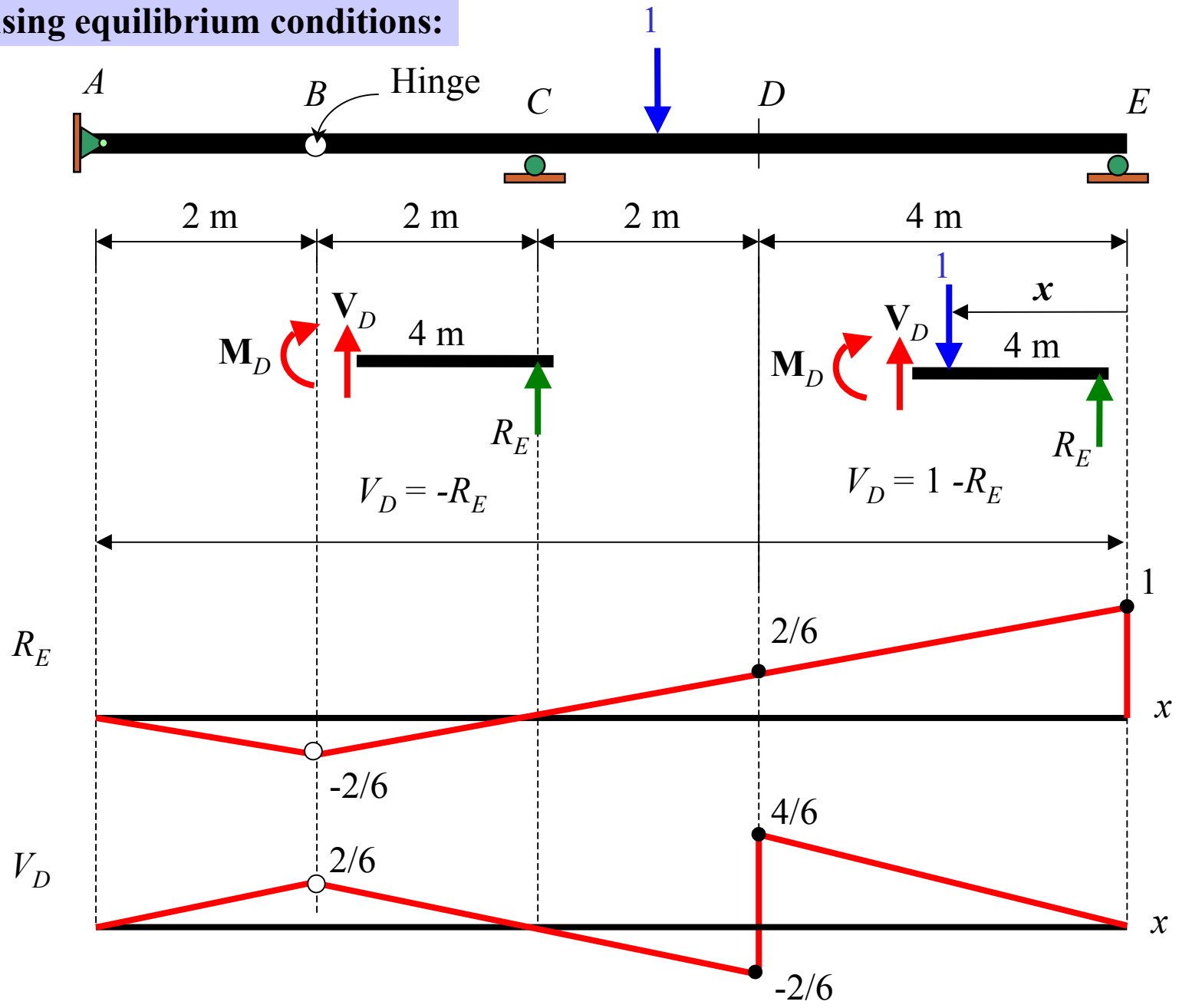

A
B
C
D
E
2 m
2 m
2 m
4 m
$\mathbf{R}_C$
R_C
8/6
1
4/6
x

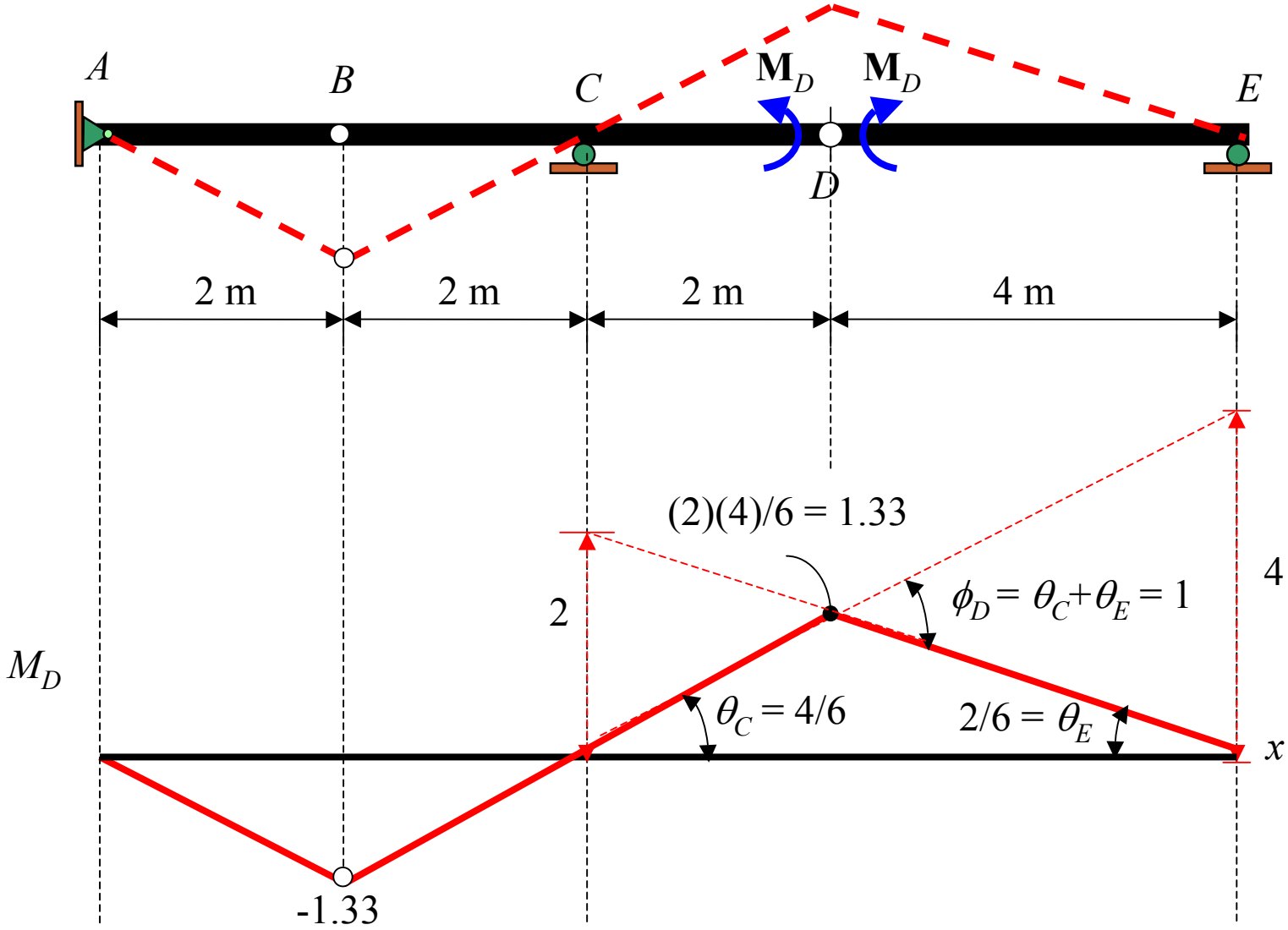

A
B
C
D
E
2 m
2 m
2 m
4 m
$\mathbf{R}_E$
1
2/6
R_E
x
-2/6



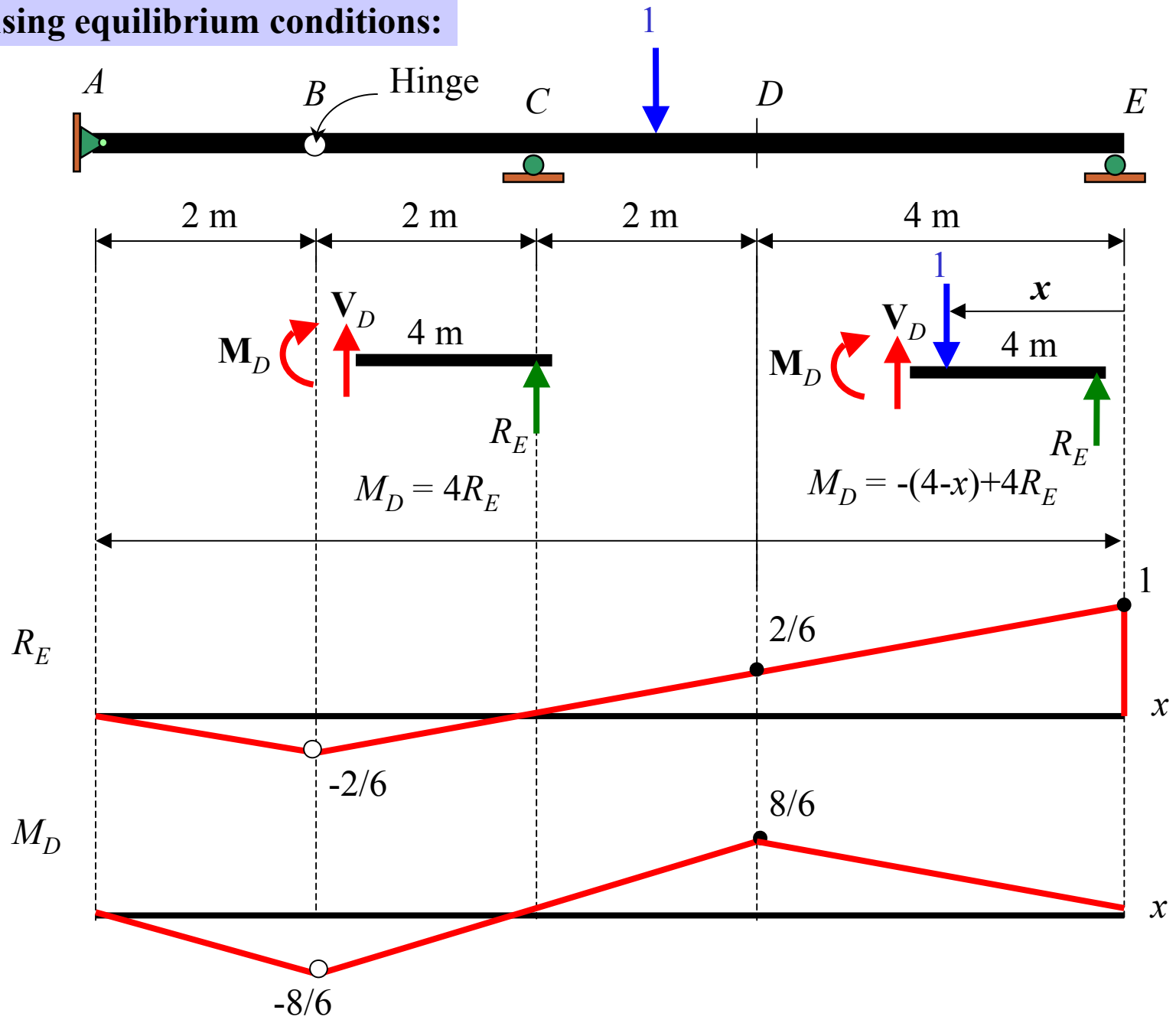
- $s_E = s_C$

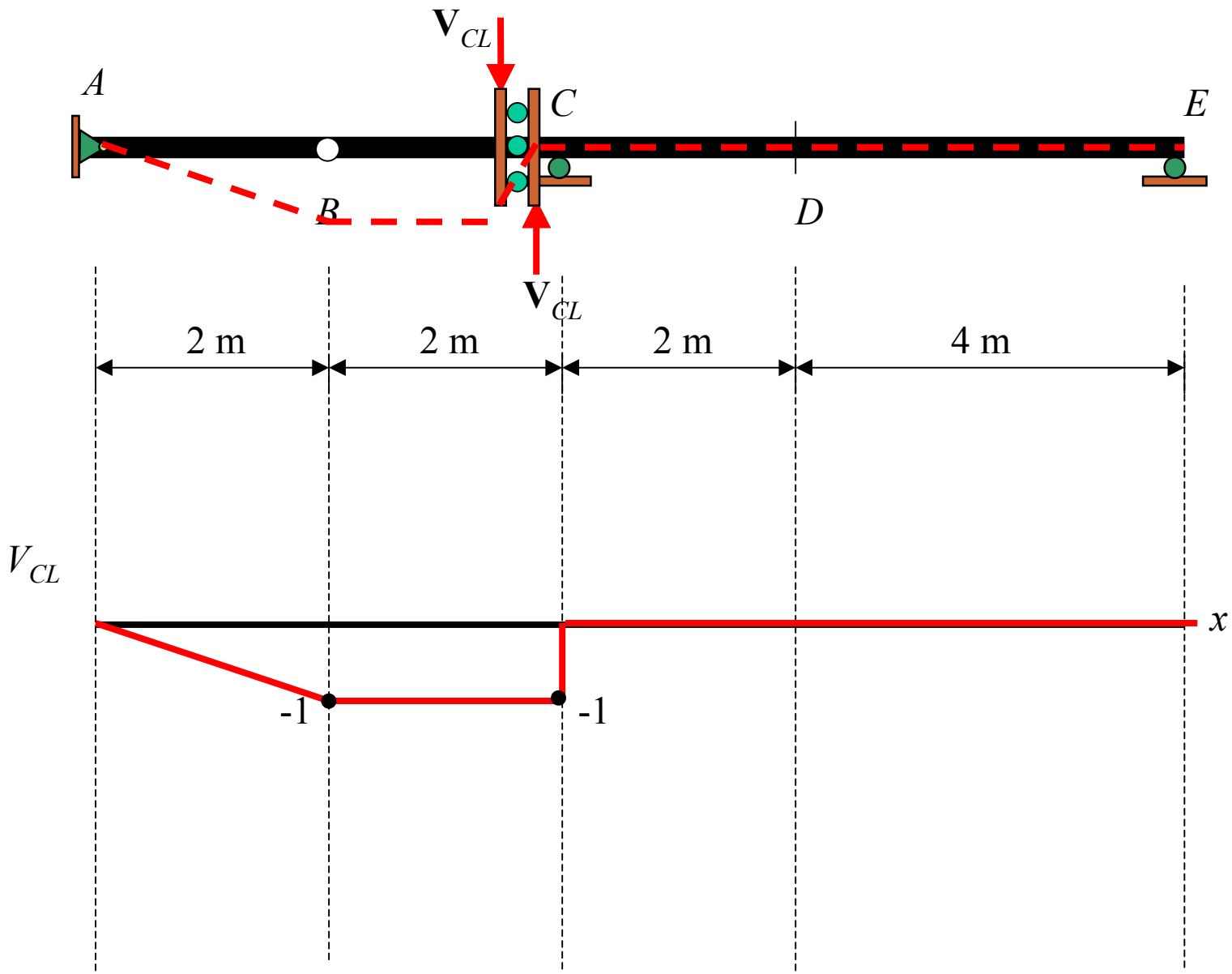
Or using equilibrium conditions:



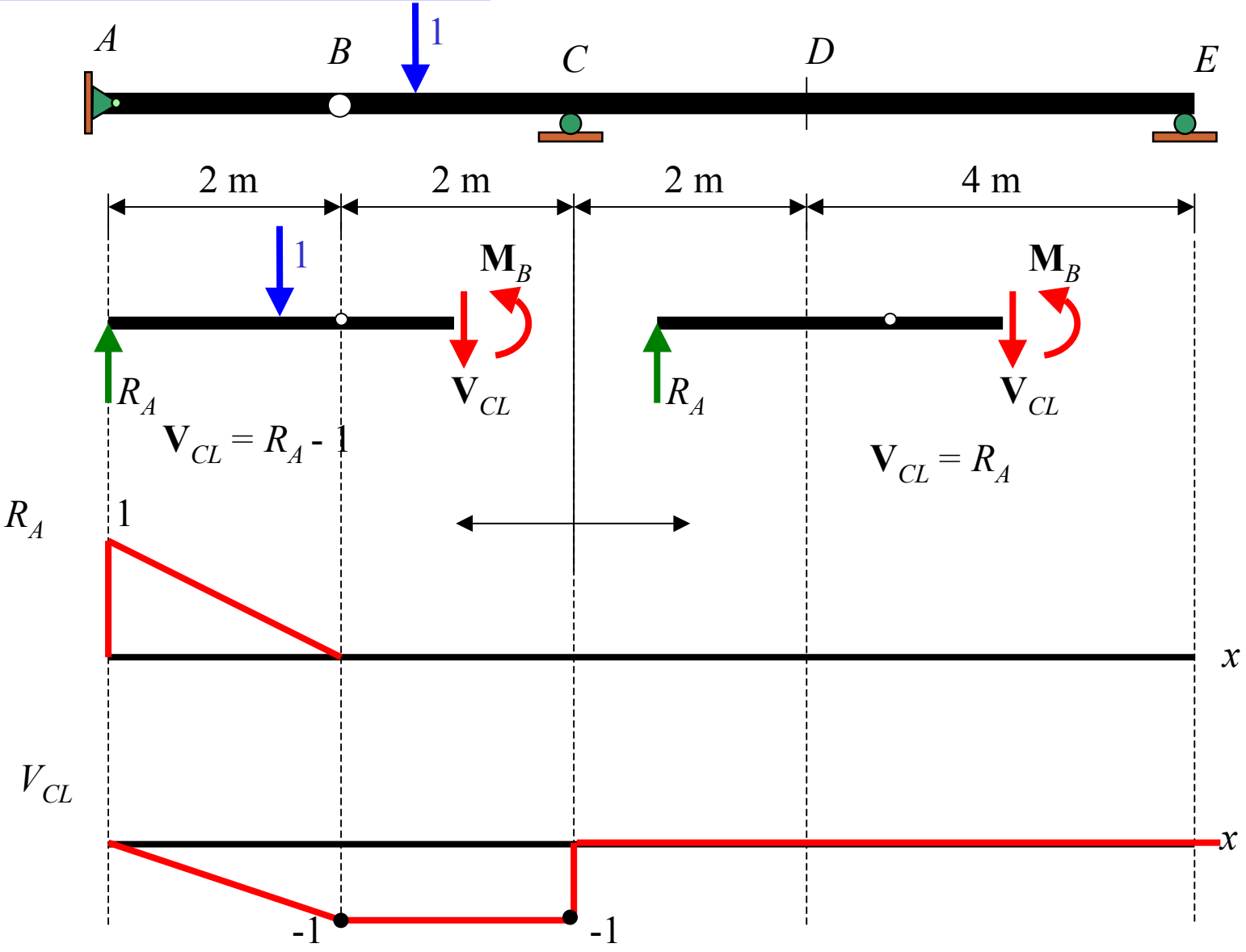

A
B
C
D
E
$\mathbf{M}_D$
$\mathbf{M}_D$
2 m
2 m
2 m
4 m
(2)(4)/6 = 1.33
2
4
M_D
$\phi_D = \theta_C + \theta_E = 1$
$\theta_C = 4/6$
$2/6 = \theta_E$
x
-1.33

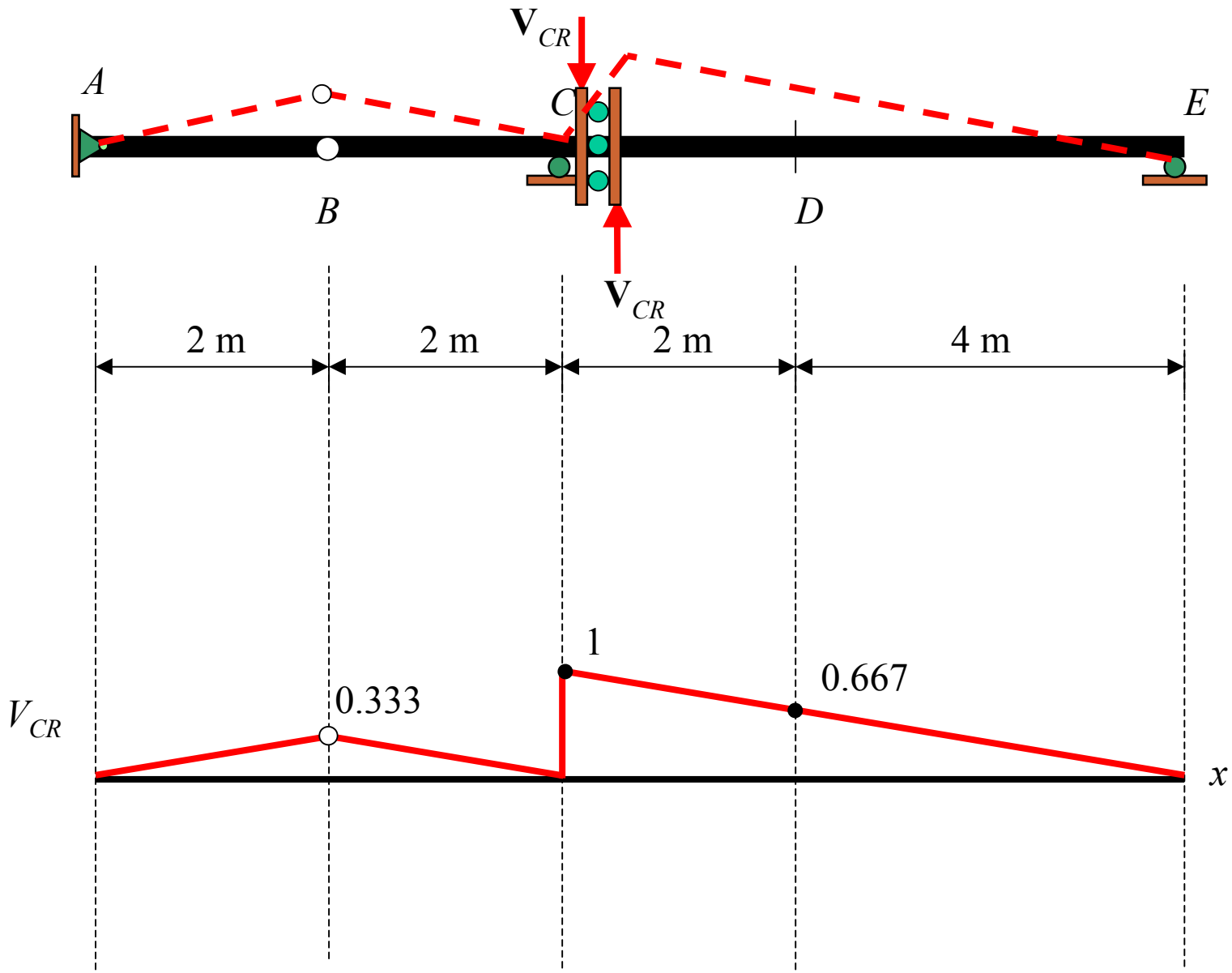
Or using equilibrium conditions:



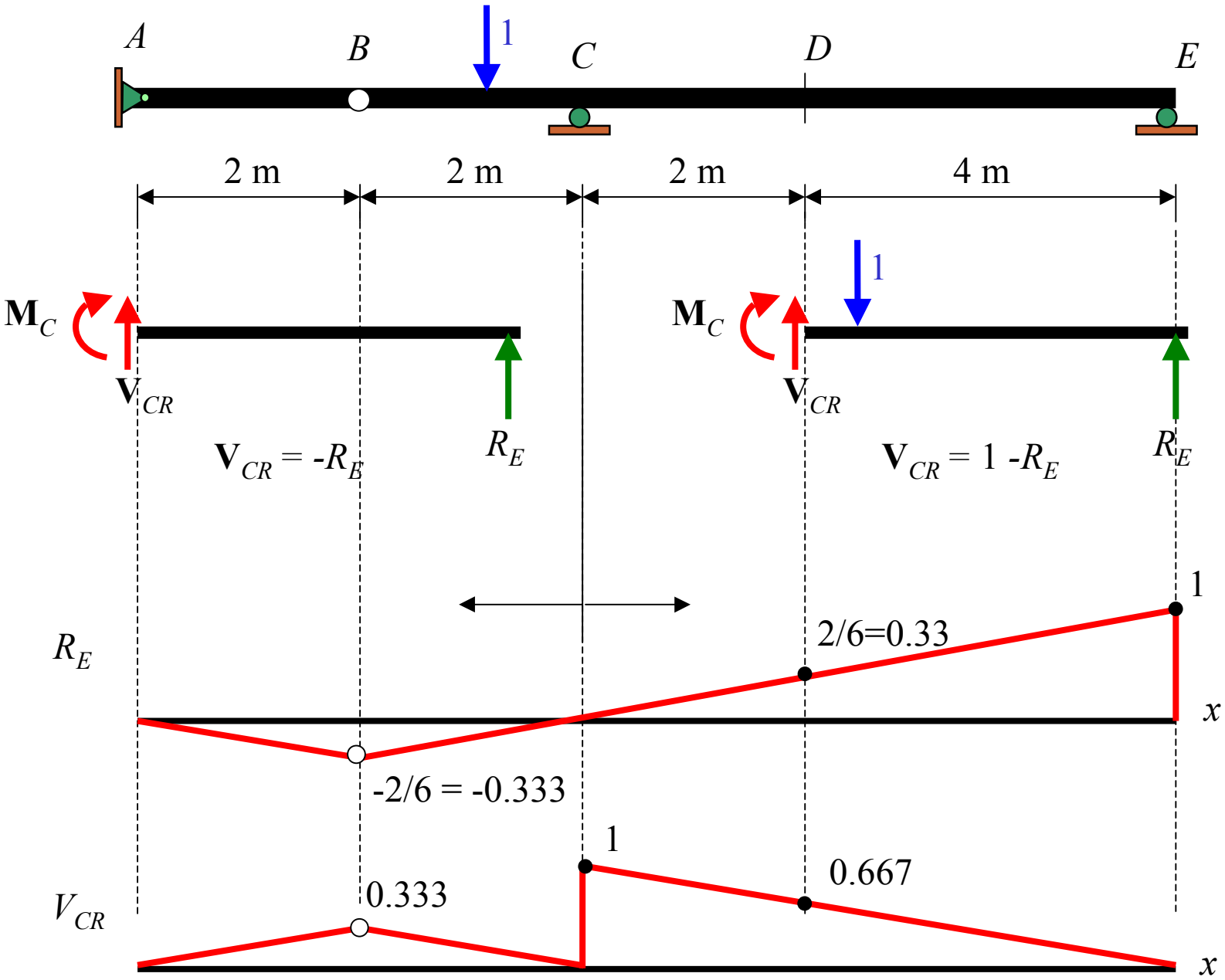

V_CL
A
C
E
B
D
V_CL
2 m
2 m
2 m
4 m
V_CL
x
-1
-1

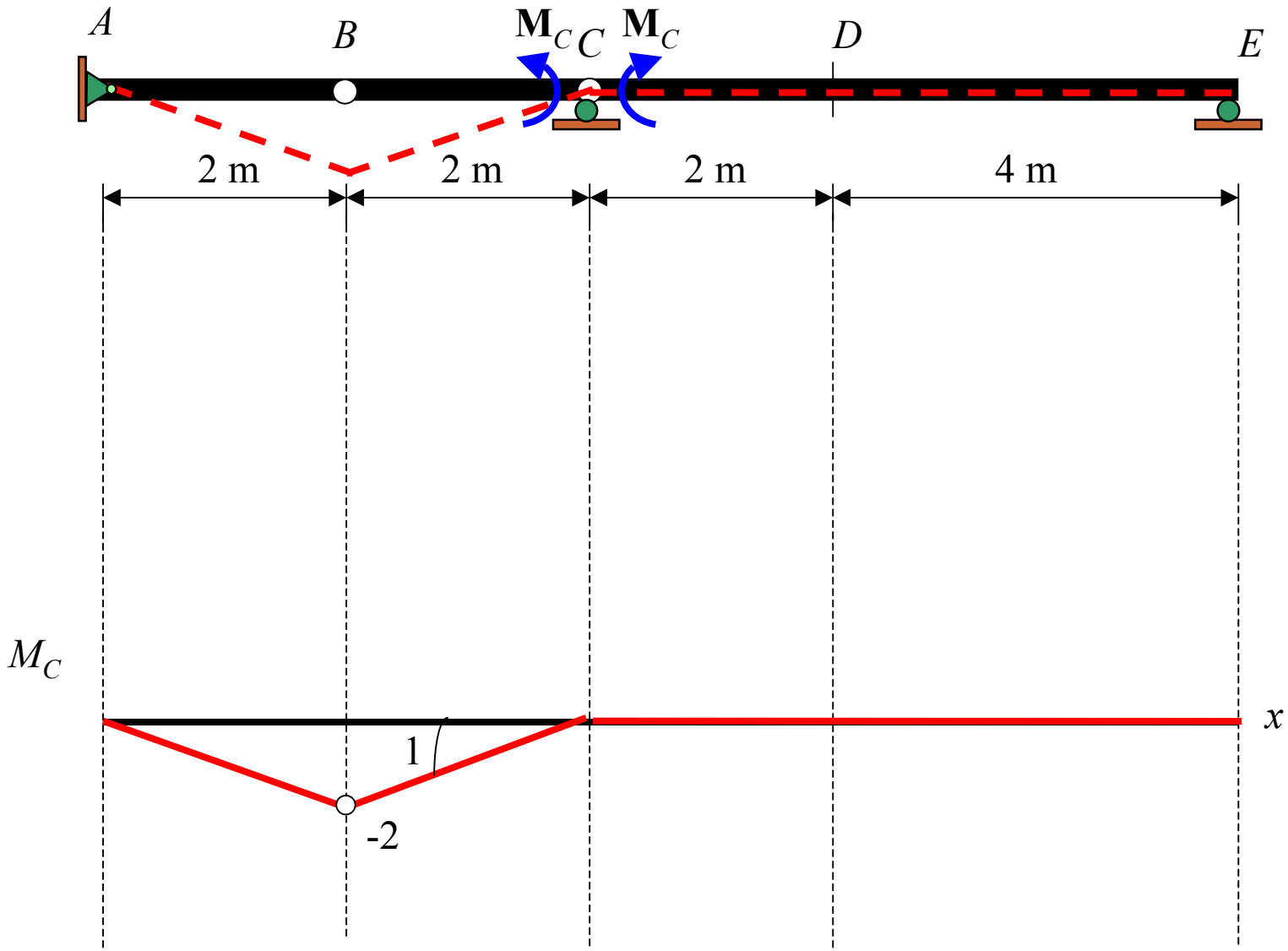
Or using equilibrium conditions:



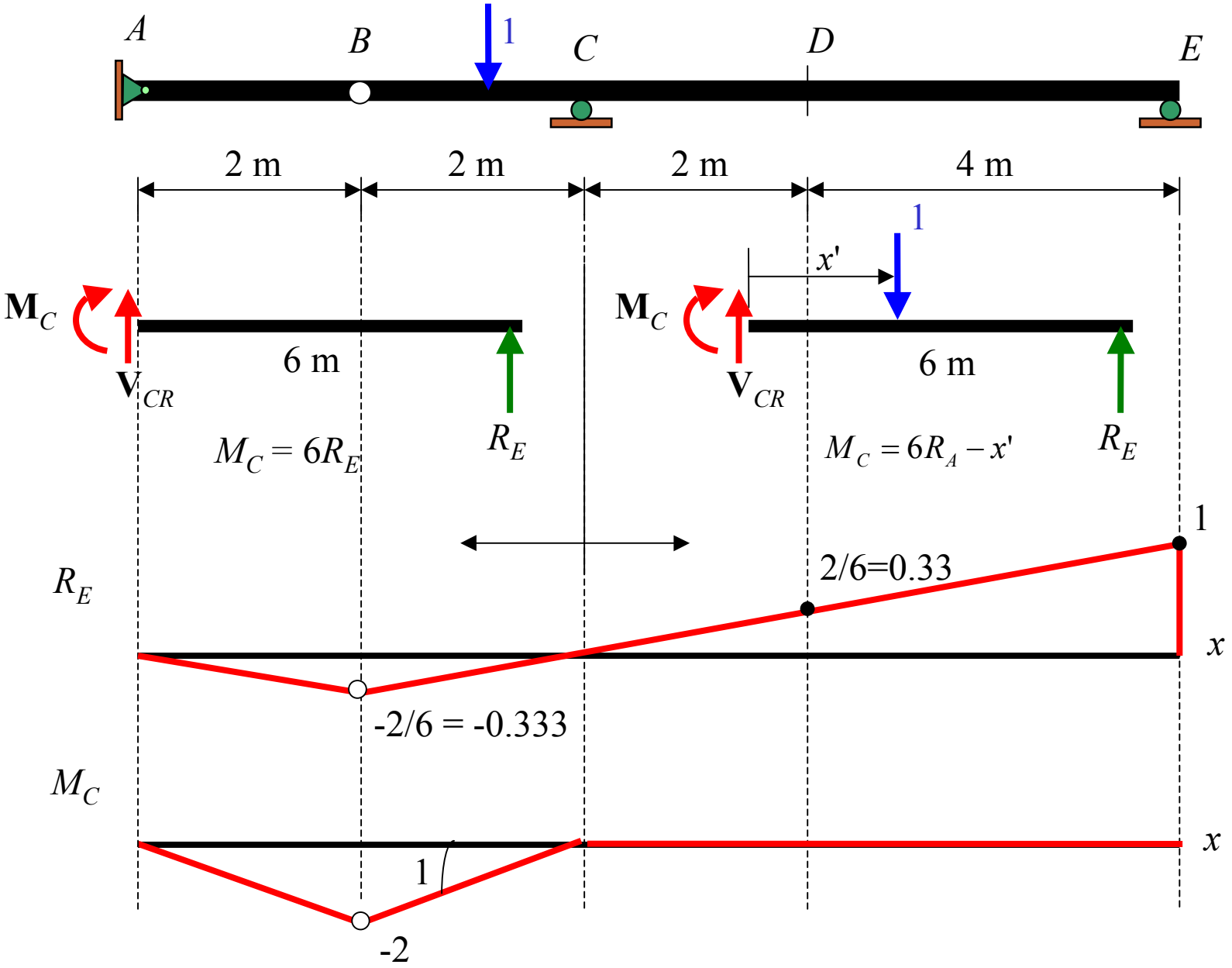

$\mathbf{V}_{CR}$
A
C
E
B
D
$\mathbf{V}_{CR}$
2 m
2 m
2 m
4 m
1
0.667
0.333
V_{CR}
x

Or using equilibrium conditions:




A
B
$\mathbf{M}_C$ C
$\mathbf{M}_C$
D
E
2 m
2 m
2 m
4 m
M_C
x
1
-2

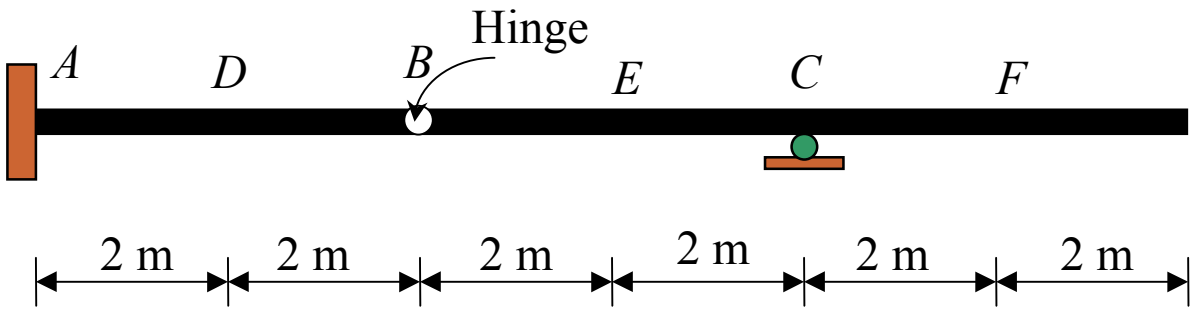
Or using equilibrium conditions:



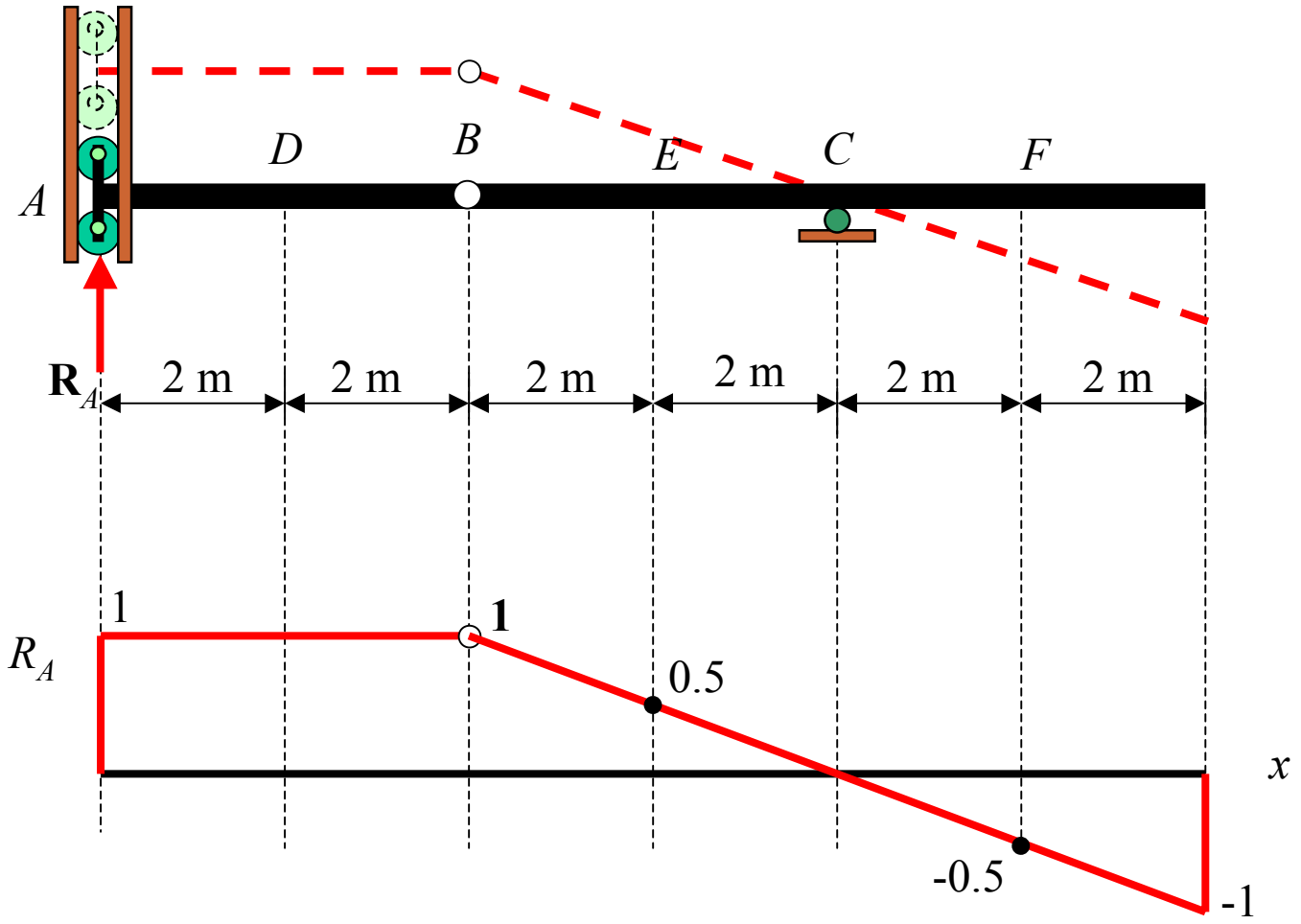
Example 6-3

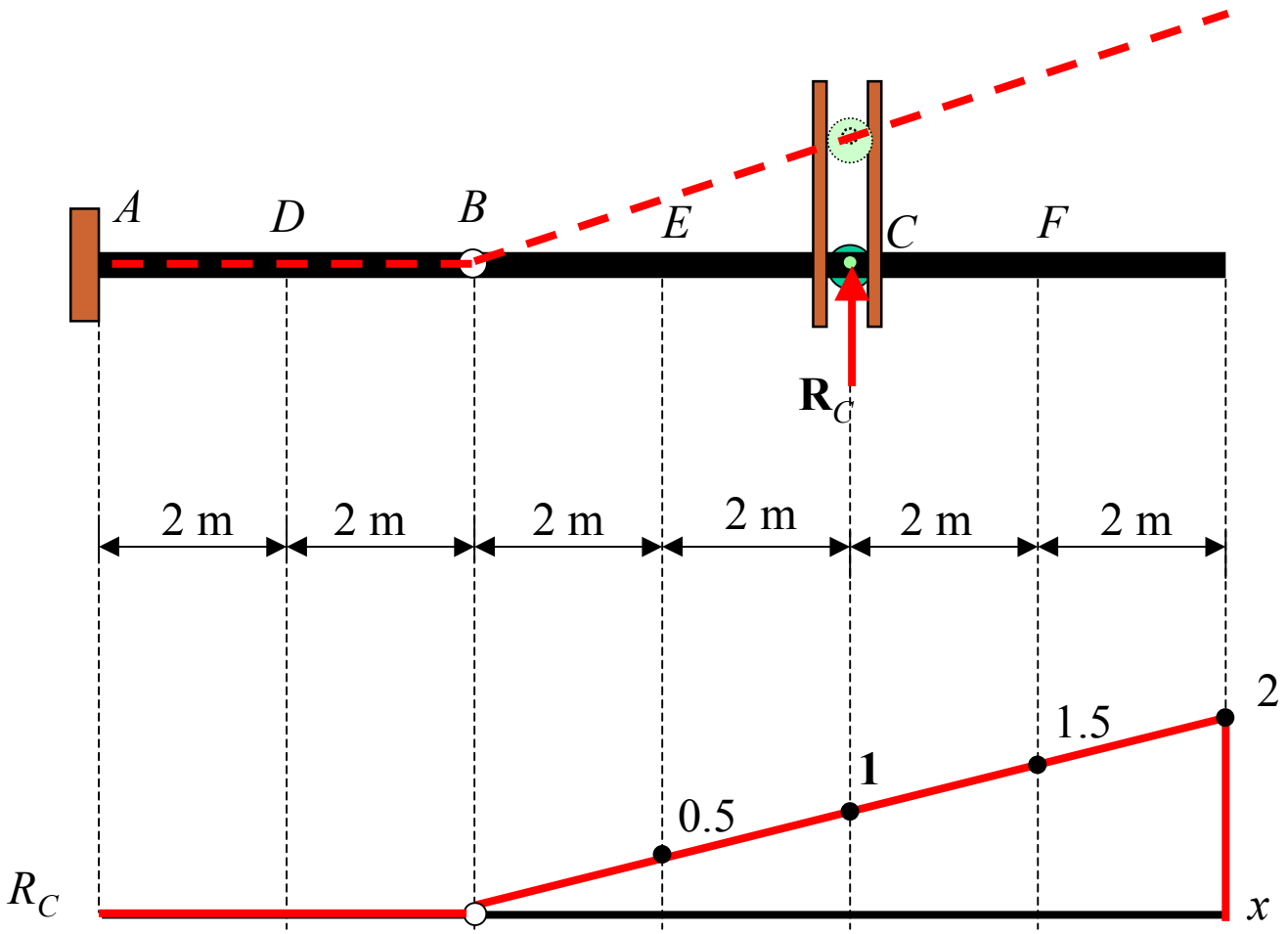
Construct the influence line for

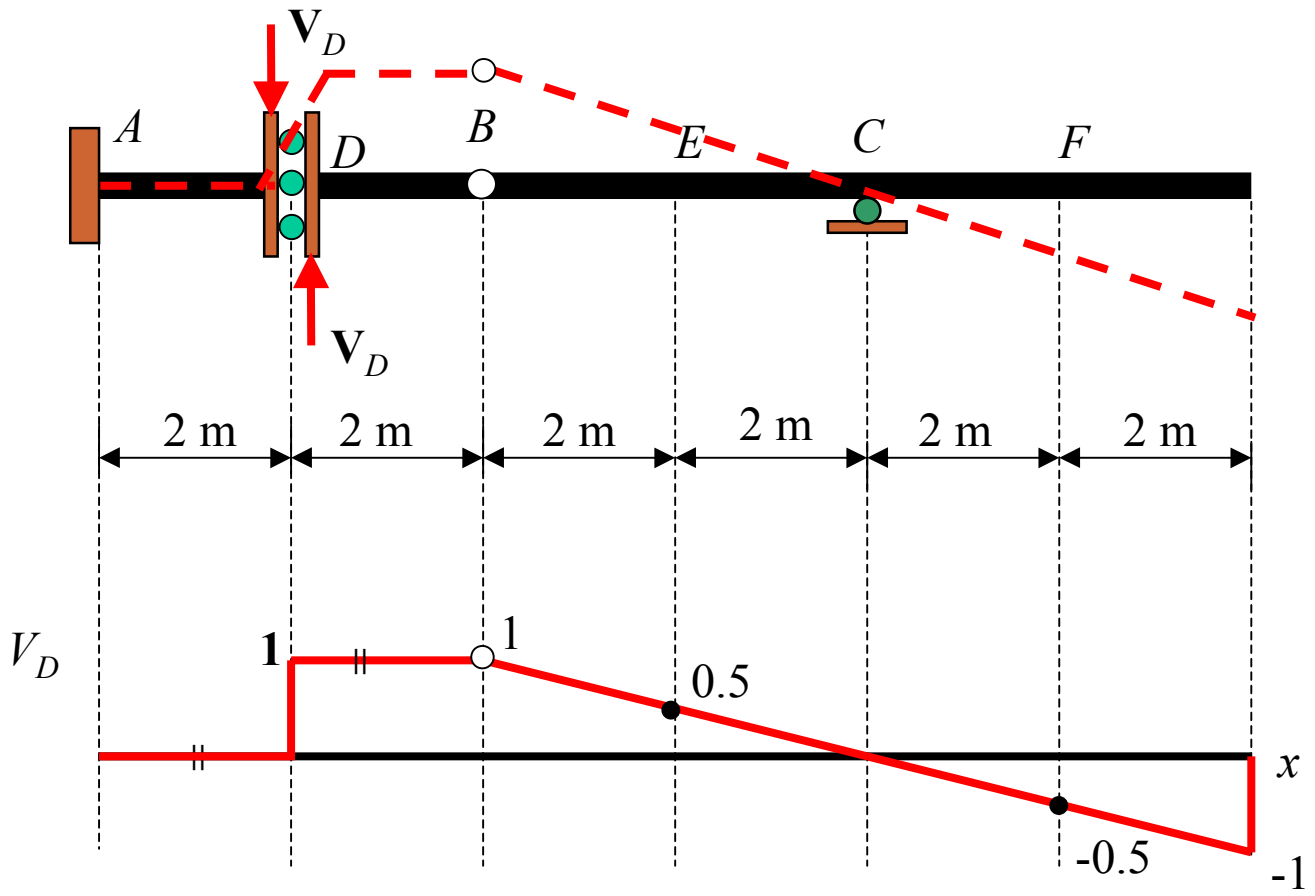
- the reaction at *A* and *C*
- shear at *D, E* and *F*
- the moment at *D, E* and *F*

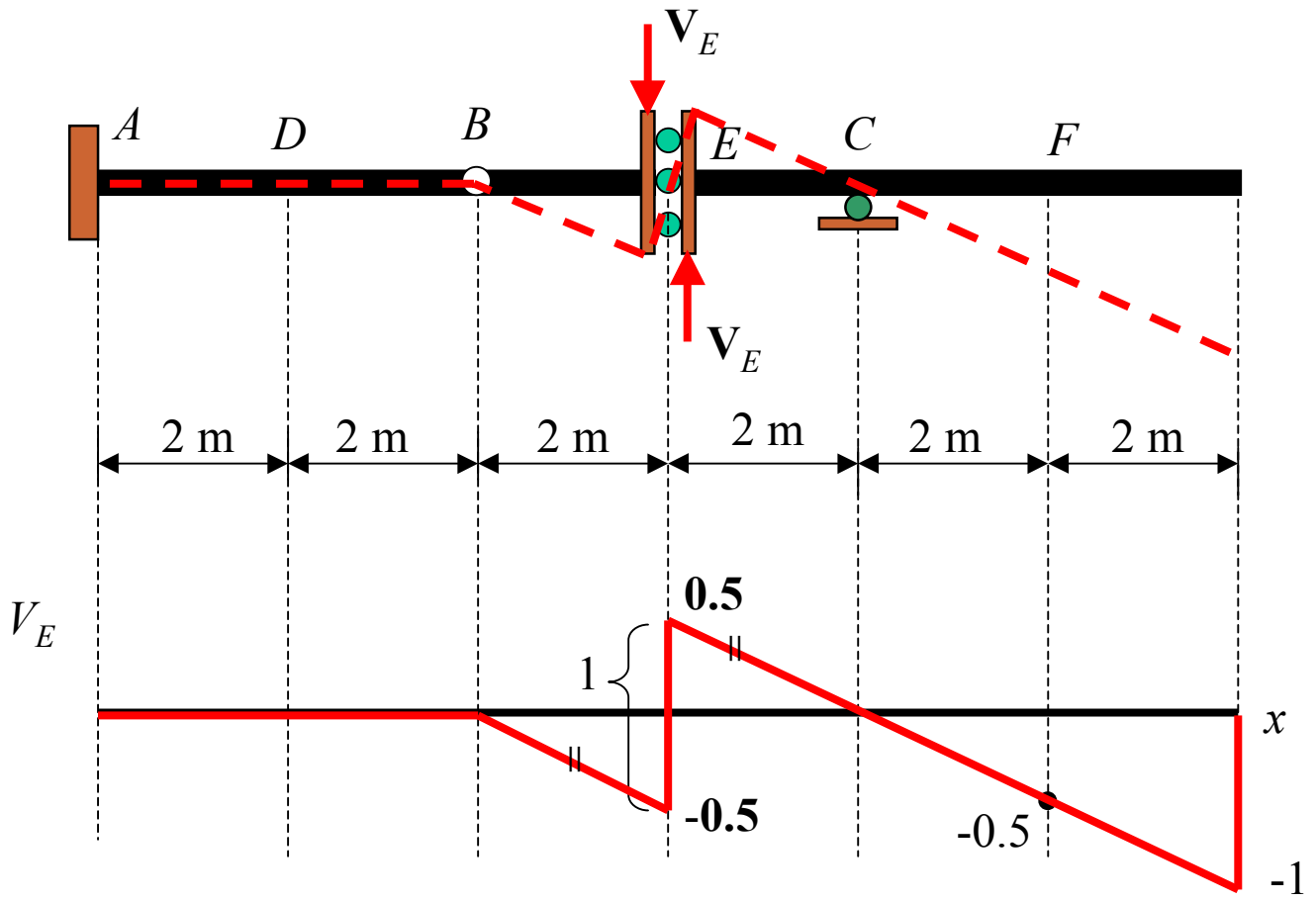


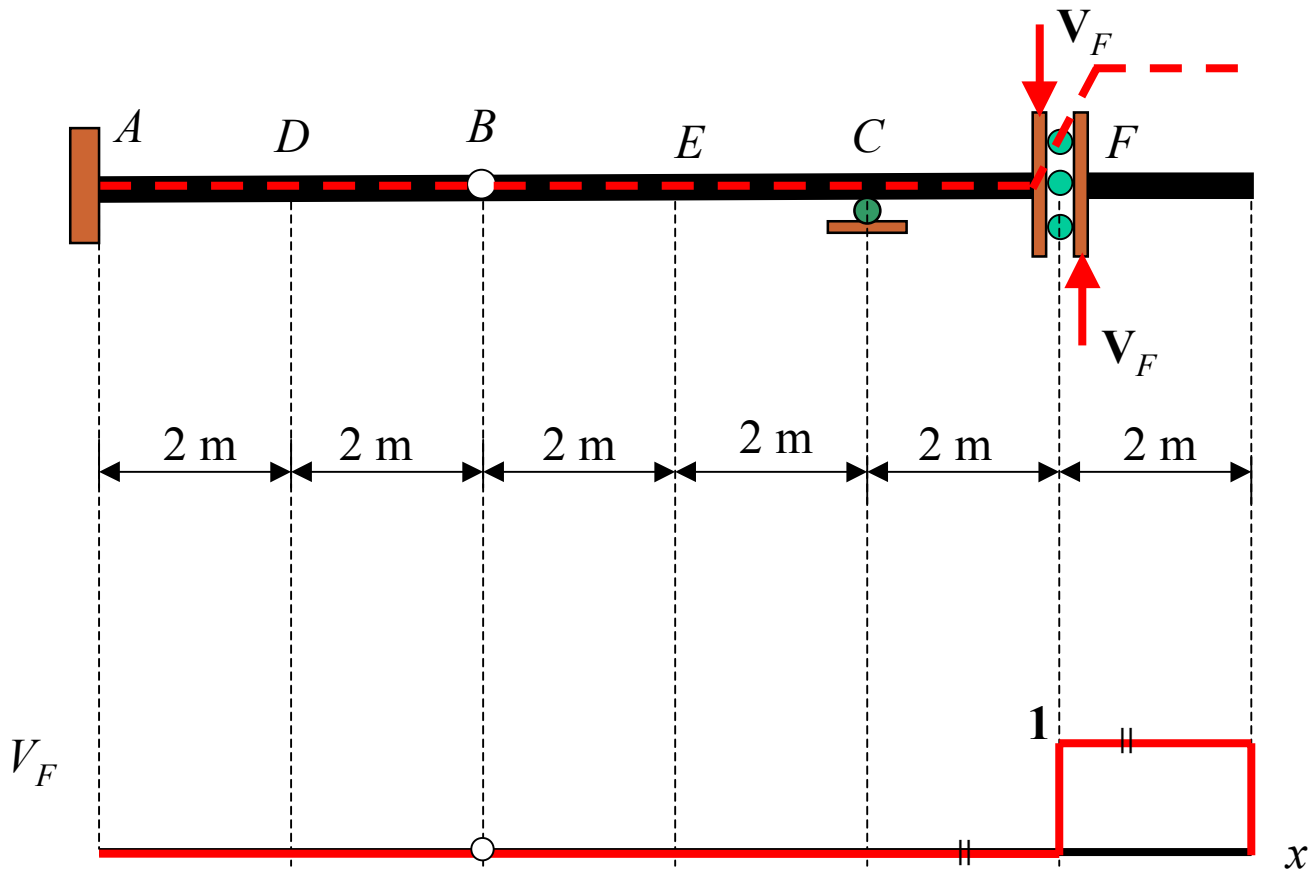
SOLUTION

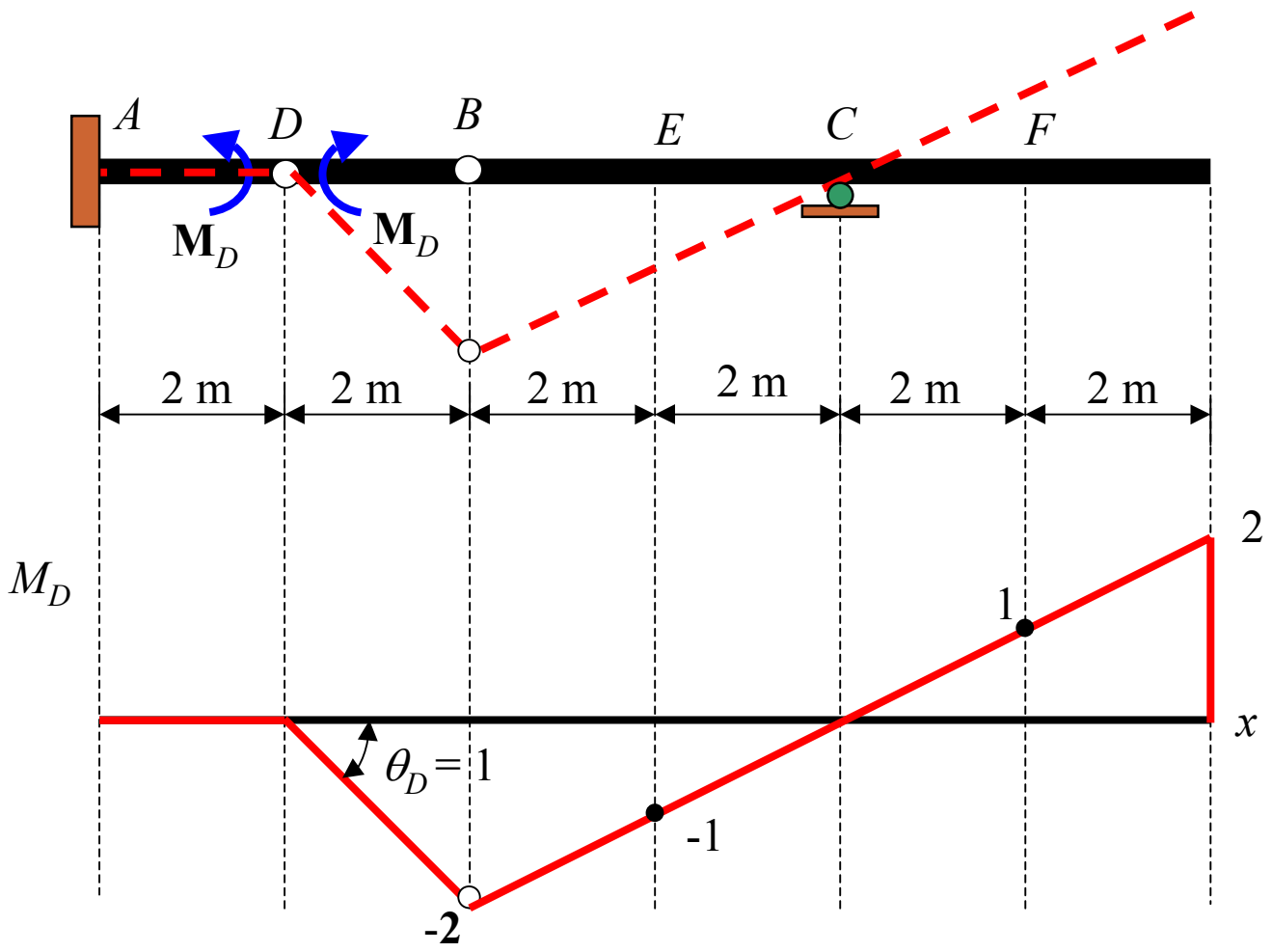


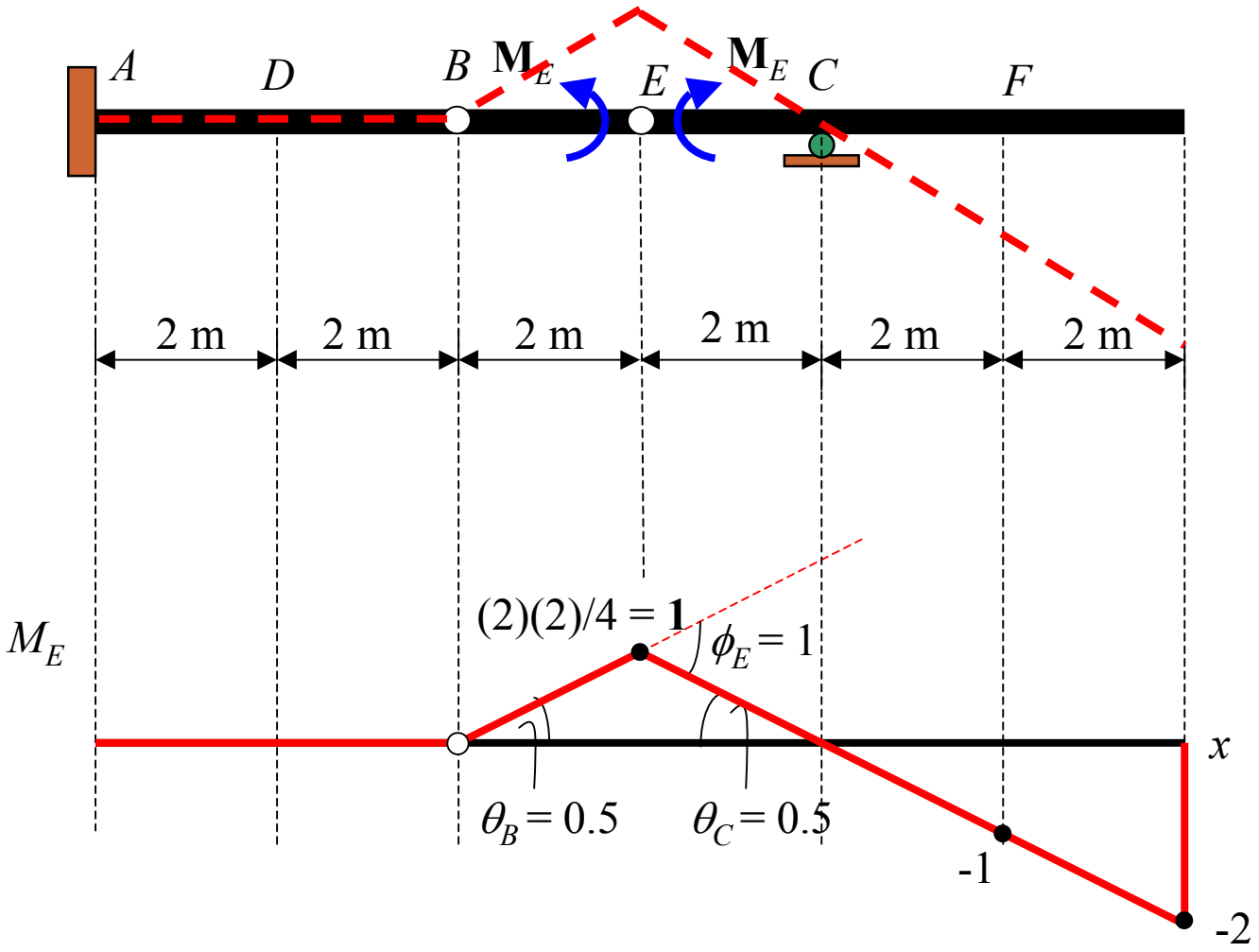

A
D
B
E
C
F
$\mathbf{R}_C$
2 m
2 m
2 m
2 m
2 m
2 m
2
1.5
1
0.5
R_C
x

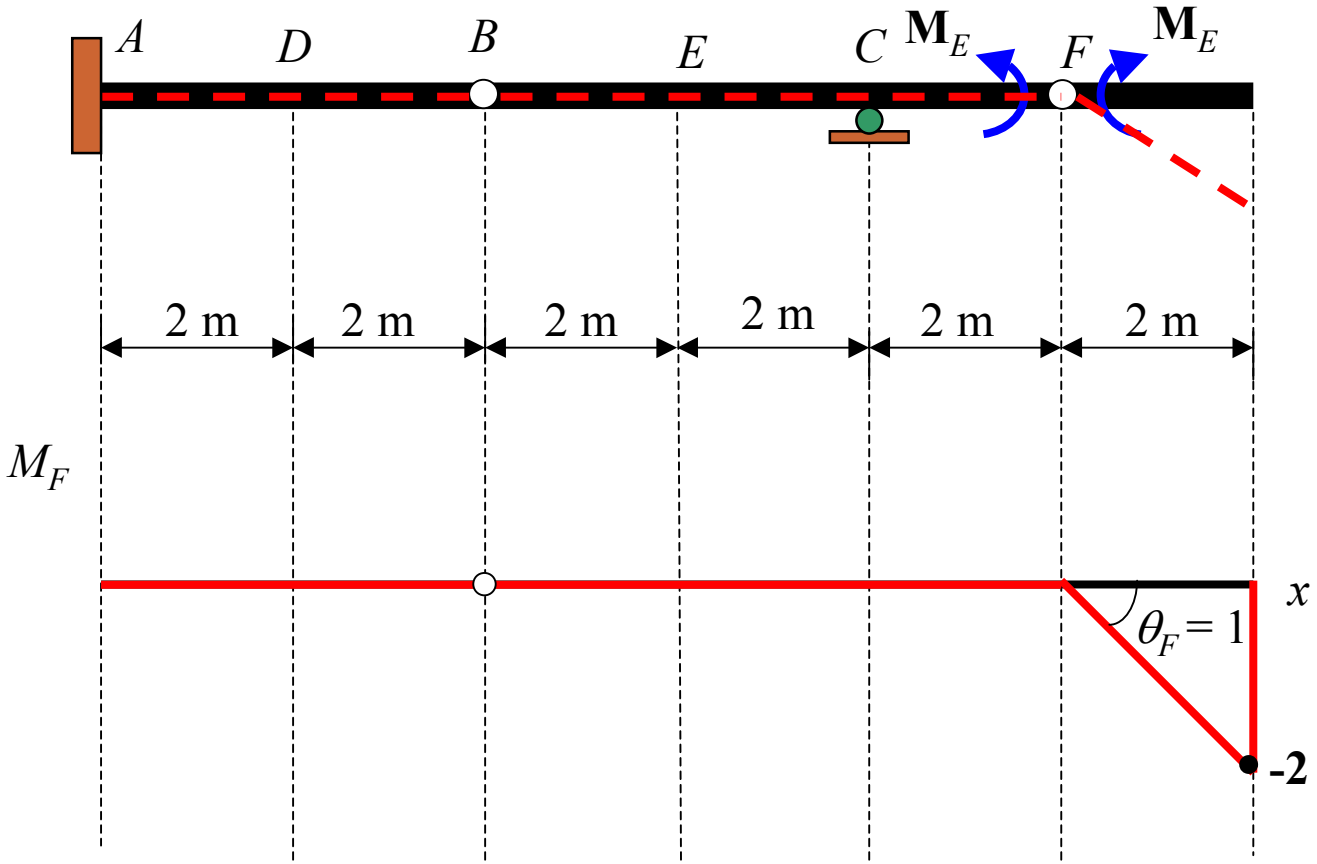

V_D
A
D
B
E
C
F
V_D
2 m
2 m
2 m
2 m
2 m
2 m
V_D
1
1
0.5
-0.5
-1
x


V_E
A
D
B
E
C
F
V_E
2 m
2 m
2 m
2 m
2 m
2 m
V_E
0.5
1
x
-0.5
-0.5
-1


A
D
B
E
C
F
$\mathbf{V}_F$
$\mathbf{V}_F$
2 m
2 m
2 m
2 m
2 m
2 m
V_F
1
x


A
D
B
E
C
F
$\mathbf{M}_D$
$\mathbf{M}_D$
2 m
2 m
2 m
2 m
2 m
2 m
M_D
2
1
$\theta_D = 1$
-1
-2
x


A
D
B
$\mathbf{M}_E$
E
$\mathbf{M}_E$
C
F
2 m
2 m
2 m
2 m
2 m
2 m
M_E
(2)(2)/4 = **1**
$\phi_E = 1$
x
$\theta_B = 0.5$
$\theta_C = 0.5$
-1
-2


A
D
B
E
C
$\mathbf{M}_E$
F
$\mathbf{M}_E$
2 m
2 m
2 m
2 m
2 m
2 m
M_F
x
$\theta_F = 1$
-2

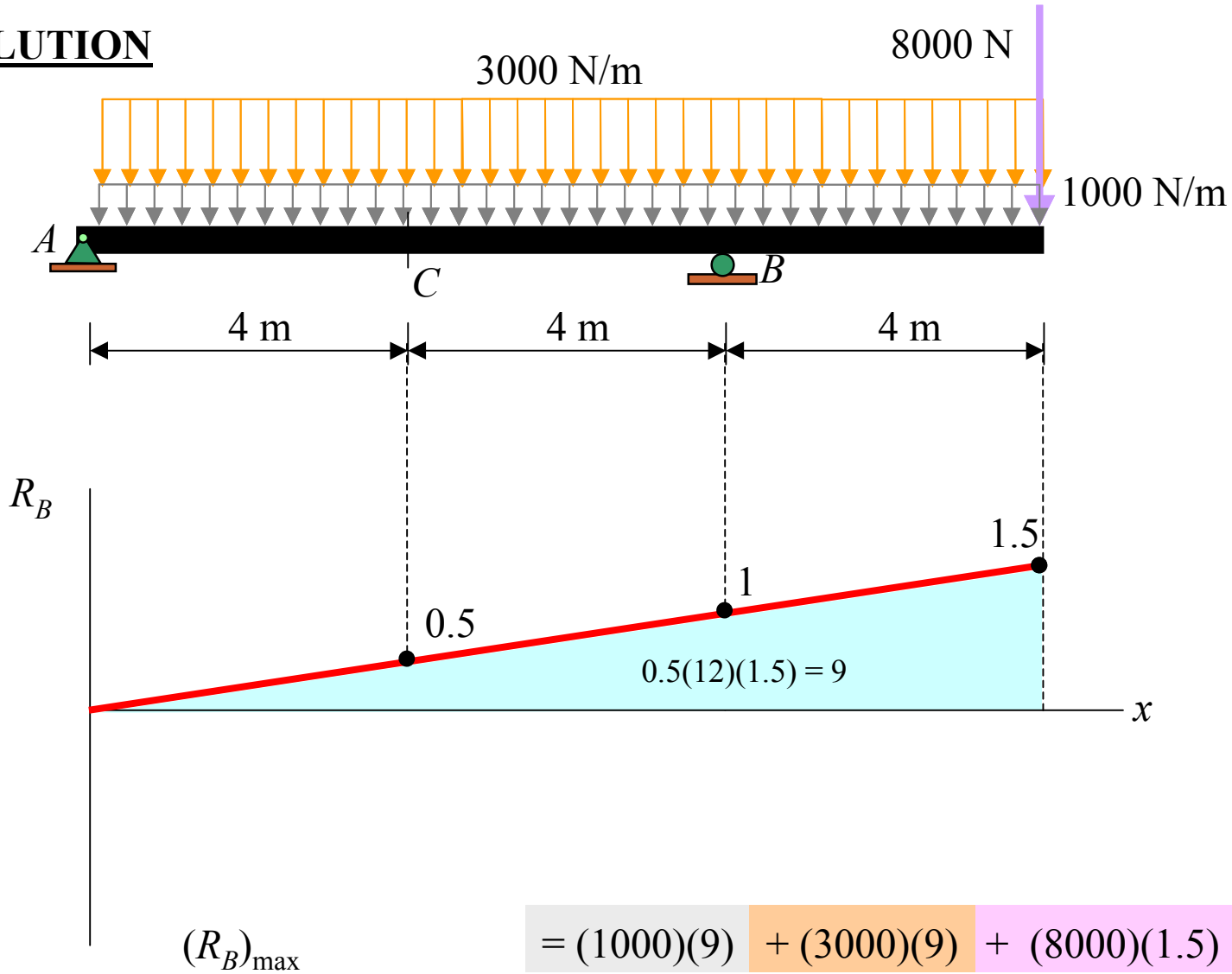
Example 6-4

Determine the maximum reaction at support *B*, the maximum shear at point *C* and the maximum **positive** moment that can be developed at point *C* on the beam shown due to

- a single concentrate live load of 8000 N
- a uniform live load of 3000 N/m
- a beam weight (dead load) of 1000 N/m

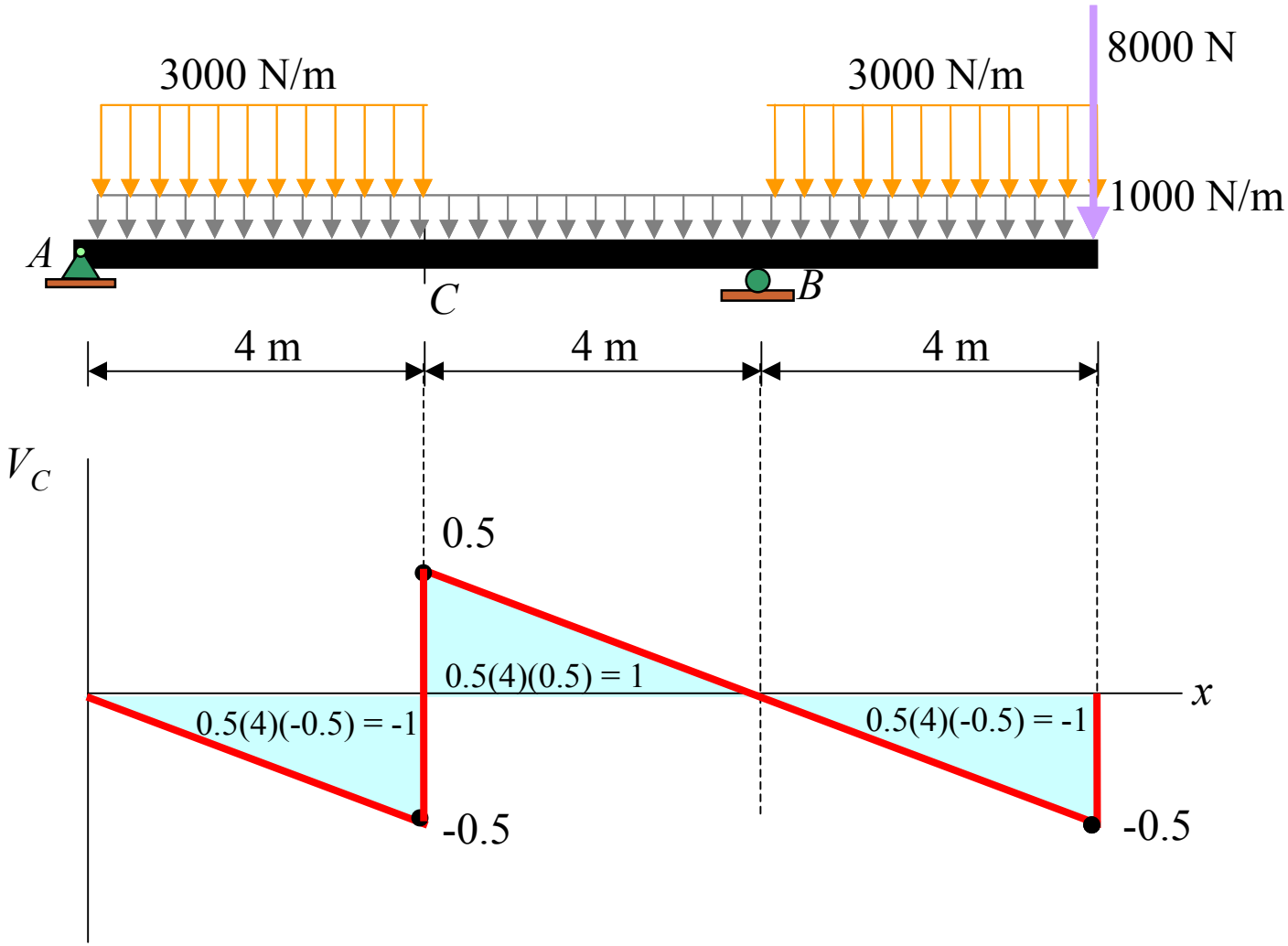


SOLUTION



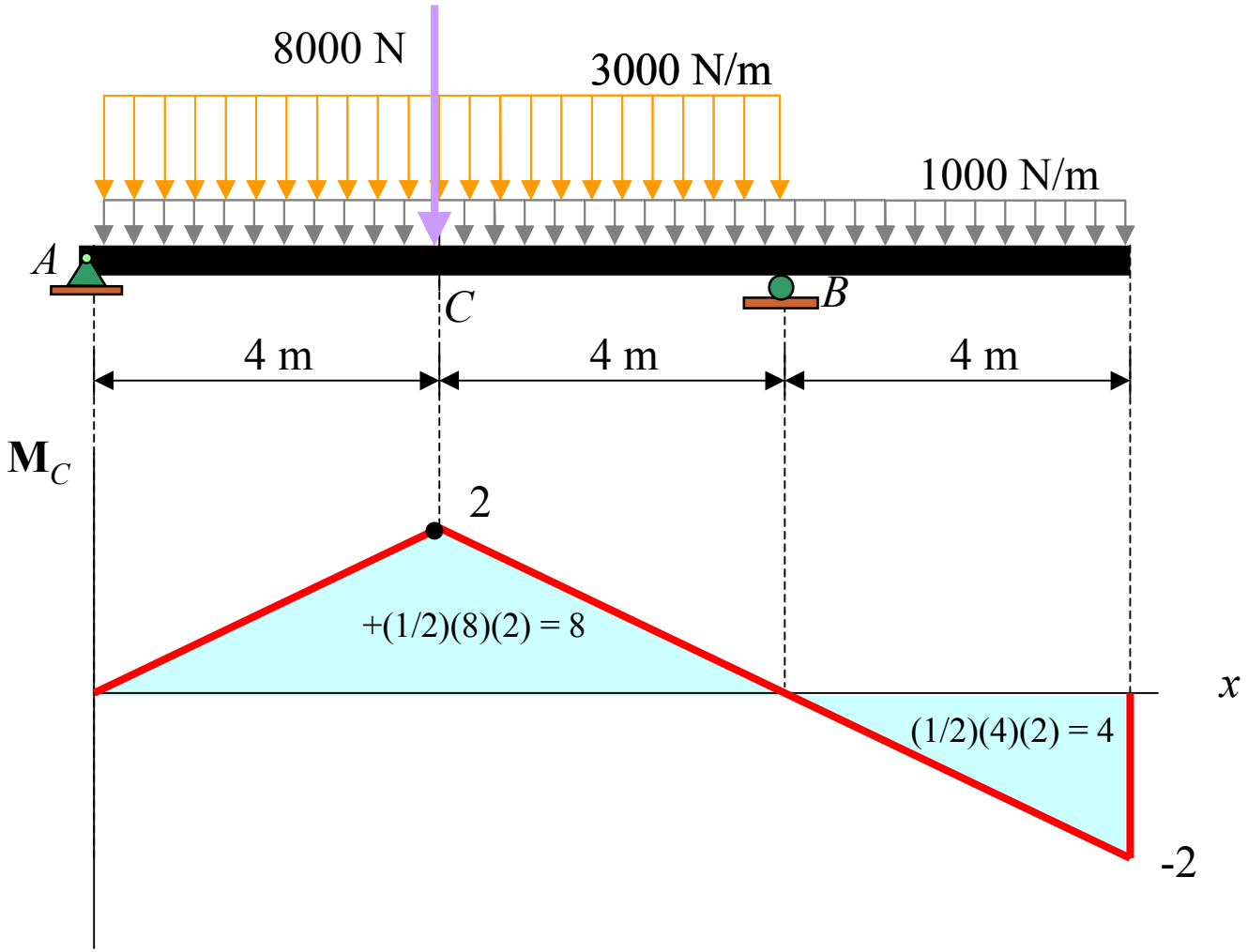
$$(R_B)_{\max} = (1000)(9) + (3000)(9) + (8000)(1.5)$$

$$= 48000 \text{ N} = 48 \text{ kN}$$



$$(V_C)_{max} = (1000)(-2+1) + (3000)(-2) + (8000)(-0.5)$$

$$= -11000 \text{ N} = 11 \text{ kN}$$



$(M_C)_{\text{max positive}}$ = (8000)(2) + (3000)(8) + (8-4)(1000)

= 44000 N•m = 44 kN•m